Stellar aberration

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December 11, 2010

1 Stellar aberration

Stellar aberration is the phenomenon that the position of a star appears to change throughout the year.

Let's see what the Galilean and the Lorentz transforms predict this change should be.

We will answer the following question:
What will the difference in the direction to a star near the ecliptic north pole be when it is observed twice with half a year between the observations?

Given an inertial frame of reference K, where an observer on the Earth at some time is instantly at rest. Let the x-axis be tangential to Earth's orbit around the Sun, and let the positive direction of the axis be opposite to the orbital velocity of the Earth. Let the y-axis point towards the ecliptic north pole.

Given a second inertial frame of reference K', where an observer on the Earth half a year later is instantly at rest. Let the x'-axis be tangential to Earth's orbit around the Sun, and let the positive direction of the axis be along the orbital velocity of the Earth. Let the y'-axis point towards the ecliptic north pole.

Note that the axes of K and K' are parallel, and that K' is moving at the speed $2v$ in the positive x-direction relative to K, where v is the orbital speed of the Earth.

Let us consider a light beam from the star that hits the origin of K, and let us assume that the observed star seems to be at the ecliptic north pole, that is, the beam is along the y-axis.

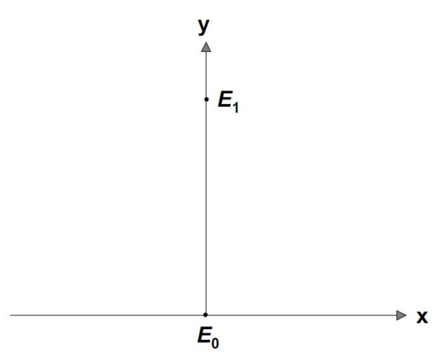

Figure 1: *Frame K*

Let E_0 be the event that the beam hits the origin, and let E_1 be an arbitrary event on the beam.

The coordinates of these events in K are:

$$
\begin{aligned}
&E_0\!: \quad t_0 = 0, \qquad x_0 = 0, \qquad y_0 = 0 \\
&E_1\!: \quad t_1 = -\frac{d}{c}, \quad x_1 = 0, \qquad y_1 = d \quad \text{where } d \text{ is an arbitrary distance}
\end{aligned}
$$

Let us transform these events to the frame K'.

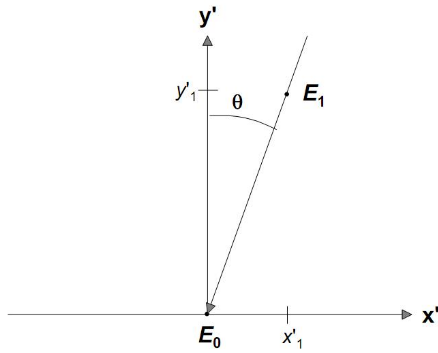

Figure 2: *Frame* K'

2 Transformed according to the Lorentz transform

The coordinates of the events in K' will be:

$$
\begin{aligned}
&E_0: \quad t'_0 = 0, && x'_0 = 0, && y'_0 = 0 \\
&E_1: \quad t'_1 = \frac{t_1 - \frac{x_1 \cdot 2v}{c^2}}{\sqrt{1-\left(\frac{2v}{c}\right)^2}} = -\frac{d}{c\sqrt{1-\frac{4v^2}{c^2}}}, && x'_1 = \frac{x_1 - 2vt_1}{\sqrt{1-\left(\frac{2v}{c}\right)^2}} = \frac{2vd}{c\sqrt{1-\frac{4v^2}{c^2}}}, && y'_1 = y_1 = d
\end{aligned}
$$

We then have:

$$
\sin\theta = \frac{x'_1}{\sqrt{x_1'^2 + y_1'^2}} = \frac{\frac{2vd}{c\sqrt{1-\frac{4v^2}{c^2}}}}{\sqrt{\left(\frac{2vd}{c\sqrt{1-\frac{4v^2}{c^2}}}\right)^2 + d^2}} = \frac{2v}{c} \tag{1}
$$

$$
\theta = \arcsin\left(\frac{2v}{c}\right) \approx \frac{2v}{c} \quad \textit{when} \quad v \ll c \tag{2}
$$

Since the mean orbital velocity of the Earth is $v = 29.78\,km/s$, and the speed of light is $c = 299792458\,m/s$, $\theta \approx \frac{2v}{c} \approx 1.987 \cdot 10^{-4}\,radians \approx 40.98\,arcseconds$.

3 Transformed according to the Galilean transform

The coordinates of the events in K' will be:

$$
\begin{array}{llll}
E_0: & t'_0 = 0, & x'_0 = 0, & y'_0 = 0 \\
E_1: & t'_1 = t_1 = -\dfrac{d}{c}, & x'_1 = x_1 - 2vt_1 = \dfrac{2vd}{c}, & y'_1 = y_1 = d
\end{array}
$$

We then have:

$$\tan\theta = \frac{x'_1}{y'_1} = \frac{\frac{2vd}{c}}{d} = \frac{2v}{c} \tag{3}$$

$$\theta = \arctan\left(\frac{2v}{c}\right) \approx \frac{2v}{c} \quad when \quad v \ll c \tag{4}$$

Since the mean orbital velocity of the Earth is $v = 29.78\,km/s$, and the speed of light is $c = 299792458\,m/s$, $\theta \approx \dfrac{2v}{c} \approx 1.987 \cdot 10^{-4}\,radians \approx 40.98\,arcseconds$.

4 Discussion

We can conclude that when an observer on the Earth observes a star near the ecliptic pole twice, with half a year between the observations, the apparent position of the star will change by 40.98 arcseconds. This is in accordance with observations.

Note that the speed 2v is the change in the magnitude of the difference of the velocities of the Earth at the two times of observation. So it is the change in the velocity of the Earth that is significant. I think the reader will understand from the symmetry of the problem that a star near the ecliptic pole during one year will appear to move in a circle with diameter 40.98 arcseconds. A star at the ecliptic will appear to move back and forth along a line which is 40.98 arcseconds long. Generally, a star will appear to move along an ellipse where the major axis is 40.98 arcseconds. The centre of this ellipse will be the position recorded as the position of the star in the star catalogues.

Note that the velocity of the star does not contribute to the stellar aberration. This is experimentally proven by the fact that the aberration of both components of binary stars are exactly the same, even if the transverse velocities of the components may be very high and in opposite directions.

However, if a star has a very high transverse velocity the observed position of the star will obviously change with time. The change in the position of the star (as defined above) is called the proper motion of the star. So the star will appear to move along an ellipse which is moving with the star's proper motion. The latter motion is however not called stellar aberration, and when observing the star, the difference will be very obvious because stellar aberration affects all the stars in a region in exactly the same way, while proper motion will change the position of the star relative to the other stars in the region. The vast majority of stars have a proper motion much less than 1 arcsecond per year, only a very few stars have a proper motion in excess of 1 arcsecond per year, the fastest one is Barnard's star with a proper motion of 10.3 arcseconds per year.

In addition to proper motion and stellar aberration, a star may also change its apparent position due to parallax. This will affect the stars differently according to their distance from the Sun, and the change will be cyclic. No star has a parallax in excess of 1 arcsecond per year.

We can sum it up thus:

- Annual cyclic motion common to all stars in a region is caused by stellar aberration.
- Annual cyclic motion relative to the other stars in a region is caused by parallax.
- Linear motion of a star relative to the other stars in a region is caused by proper motion.