Gravitational deflection of light by the Sun

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Simulation of deflection of light by the Sun

The deflection of light by the Sun is simulated according to Newton and according to The General Theory of Relativity [GR].

The light source, a star

The light from a star will ever grace the Sun as observed from the Earth, only if the star is in, or very close to the ecliptic plane. Then the angle star-Sun as observed from the Earth will change by 360^0 as the Earth moves around the ecliptic. But since the two halves of the year are symmetric, we will simulate the deflection for the angle star-Sun varying from 0.266^0 to 179.734^0, the two angles where a light beam from the star which passes closely by the Earth will grace the Sun. The former angle is when the star from the Earth is seen at the rim of the Sun, the latter angle is when the beam from the star graces the Sun after having passed the Earth.

The simulation is done with two alternative sources; a fictive star at infinite distance in the ecliptic plane, and the star 13 Tauri which is at distance 409 light years in the ecliptic plane.

The gravitational acceleration of a photon by the Sun

We will assume that the gravitational acceleration of a photon is the same as the gravitational acceleration of a test particle with initial speed equal to c, the speed of light in vacuum.

The gravitational acceleration of a photon according to Newton

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = -\frac{GM}{r^2} \cdot \hat{r} \tag{1}$$

Where:

$\vec{v}$	= velocity of photon in solar frame	$[m/s]$
$\vec{r}$	= distance vector sun - photon	$[m]$
r	= $\lvert\vec{r}\rvert$, magnitude of $\vec{r}$	$[m]$
$\hat{r}$	= unity vector parallel to $\vec{r}$	
G	= Gravitational constant	$[m^3 \cdot kg^{-1} \cdot s^{-2}]$
M	= solar mass	$[kg]$

The gravitational acceleration of a photon according to GR

The post-Newtonian approximation of the gravitational acceleration predicted by GR is:

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = -\frac{GM}{r^2}\left(1 - 4\frac{GM}{rc^2} + \frac{v^2}{c^2}\right)\hat{r} + \frac{4GMv^2\,(\hat{r}\cdot\hat{v})}{r^2c^2}\cdot\hat{v} \tag{2}$$

Where:

$\vec{r}$	= distance vector sun - photon	$[m]$
r	= $\lvert\vec{r}\rvert$, magnitude of $\vec{r}$	$[m]$
$\hat{r}$	= unity vector parallel to $\vec{r}$	
$\vec{v}$	= velocity of photon in solar frame	$[m/s]$
v	= $\lvert\vec{v}\rvert$, magnitude of $\vec{v}$	$[m/s]$
$\hat{v}$	= unity vector parallel to $\vec{v}$	
c	= speed of light in vacuum	$[m/s]$
G	= gravitational constant	$[m^3 \cdot kg^{-1} \cdot s^{-2}]$
M	= solar mass	$[kg]$

The simulation

The path of the photon is simulated in principle from the star to infinity, in reality from 100 AU before the Sun to 100 AU after the Sun. The difference between the velocities of the photons at these points and the velocities of the photon at the star and at infinity respectively are assumed to be negligible.

The total deflection of the light is $\theta_t = |\angle\vec{v_s} - \angle\vec{v_i}|$ where $\vec{v_s}$ is the velocity of the photon at the star, and $\vec{v_i}$ is the velocity of the photon at infinity.

The deflection of the light observed from the Earth is $\theta = |\angle\vec{v_s} - \angle\vec{v_e}|$ where $\vec{v_s}$ is the velocity of the photon at the star, and $\vec{v_e}$ is the velocity of the photon as it hits/passes close by the Earth.

Predicted deflections

Predicted total deflection

$$\theta_t = (1+\gamma)\cdot\frac{2GM}{b\cdot c^2} \tag{3}$$

Where:

γ	= PPN parameter, $\gamma = 1$ means prediction is according to GR, $\gamma = 0$ means prediction is according to Newton
b	= the impact parameter, closest approach to Sun
c	= speed of light in vacuum
G	= gravitational constant
M	= solar mass

However, the impact parameter is hard to calculate, so the following approximation is used: $b \approx AU \cdot \sin\phi$ where AU is an astronomical unit (distance Sun-Earth), and ϕ is the angle star-Sun as observed from the Earth.

The total deflection can then be written:

$$\theta_t = (1+\gamma)\cdot\frac{2GM}{AU\cdot\sin\phi\cdot c^2} \tag{4}$$

Predicted deflection observed from the Earth

$$\theta = (1+\gamma) \cdot \frac{GM}{AU \cdot c^2} \cdot \frac{1+\cos\phi}{\sin\phi} \tag{5}$$

Where:

γ = PPN parameter, $\gamma = 1$ means prediction is according to GR, $\gamma = 0$ means prediction is according to Newton
AU = an astronomical unit (distance Sun-Earth)
ϕ = angle Sun-Earth as observed from the Earth
c = speed of light in vacuum
G = Gravitational constant
M = solar mass