Improved Test of General Relativity with Radio Doppler Data from the Cassini Spacecraft

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Abstract

Radio Doppler data from the Cassini spacecraft during its solar conjunction in June 2002 can be used to test General Relativity. In terms of the standard post-Newtonian parameter γ, the result is $\gamma - 1 = (-4.8 \pm 5.7) \times 10^{-5}$, including both random and systematic error. Einstein's theory has survived yet another test.

According to the theory of General Relativity (GR), a light ray propagating in the Sun's gravitational field is effectively refracted, with the vacuum index of refraction n augmented by a refractivity inversely proportional to the distance r from the Sun's center of mass. This GR contribution to the refraction is given by [1],

$$n = 1 + \frac{2GM}{c^2 r} \tag{1}$$

The proportionality constant $2GM/c^2$ is equal to the Sun's Schwarzschild radius, R_g = 2.9532 km. In this theory the gravitational field causes ray bending towards the Sun's center, one of the classical GR tests, and in addition, electromagnetic waves are time delayed and frequency shifted. Time-delay tests became feasible in the 1960s with the advent of radar ranging to planets and radio ranging to spacecraft [2]. The most accurate test is provided by radio ranging to Viking Landers on the surface of Mars. In terms of the standard post-Newtonian parameter γ, the Viking result is $\gamma = 1.000 \pm 0.002$ [3]. The physical interpretation of γ is that it measures the amount of space curvature produced by solar gravity, and its value is exactly unity in GR. When this parameterized curvature is expressed as an effective refractivity, the result is,

$$n = 1 + \frac{1+\gamma}{2}\frac{R_g}{r}. \tag{2}$$

The gravitational time delay Δt can be derived from Eq. 2 for a two-way signal transmitted from Earth (uplink), received by a spacecraft, and transmitted back to Earth (downlink). The result is,

$$\Delta t = (1+\gamma)\,\frac{R_g}{c}\ln\left(\frac{r+R+\rho}{r+R-\rho}\right). \tag{3}$$

where r is the distance from Sun to spacecraft, R is the distance from Sun to Earth, and ρ is the distance from Earth to spacecraft.

When fitting ranging data, Eq. 3 is the appropriate model to use for solar-system gravitation, although it requires a careful attention to details related to the correspondence between coordinates and measured quantities [4]. When fitting Doppler data, Eq. 3 is still appropriate, but now the Doppler data represents the change in range over some Doppler integration time T_C. Actually, as pointed out by Ciufolini and Wheeler [5], the GR time delay must always be referred to some baseline ranging measurement, hence all Shapiro time-delay tests depend on a range change. In addition, the time-delay equation is coordinate dependent. It is only valid if consistent with the coordinate locations of the receiver and transmitter.

On the other hand the Doppler frequency shift is a relativistic invariant, similar to perihelion precession or light deflection. It can be defined without reference to any particular coordinate frame [6].

Ideally, a Doppler experiment would provide continuous phase data (cycle count) for several days about conjunction, thereby providing accumulative range-change information to much better accuracy than the ranging data, which is limited by about a seven meter random error. In practice, the Cassini Doppler experiment relies on non continuous Doppler data, with each measurement integrated over an interval $T_C = 300$ s. The Doppler data is effectively a time series of frequency measurements, averaged over each T_C interval, but with both small gaps of several minutes and large gaps of about 16 hours or more. Only one station of the DSN, DSS25 at Goldstone California, is equipped with a Ka-band (34,316 MHz) transmitter, required for full Doppler accuracy, hence continuous data is impossible.

For purposes of modeling the Cassini Doppler data, one can show that the coordinate-independent GR fractional frequency change $\Delta\nu/\nu$ can be obtained by simply differentiating the coordinate-dependent Eq. 3 with respect to time t. When the ray path is near the solar limb, the GR expressions for range delay and Doppler shift, expressed in terms of the line-of-sight velocity v_r, can be approximated by,

$$\begin{aligned} \Delta t &= (1+\gamma)\frac{R_g}{c}\ln\left(\frac{4rR}{b^2}\right) \\ \Delta v_r &= -(1+\gamma)\frac{R_g}{b}\frac{db}{dt} \end{aligned} \tag{4}$$

where b is the impact parameter for the ray path. As pointed out by Iess et al. [7], the coefficient of $-db/dt$ is just the angular deflection of the ray path as derived from Eq. 2. When thought of as a series of frequency measurements, a Doppler experiment has more in common with light deflection than it does with Shapiro time delay. In fact, it is a cleaner test of GR than the ranging tests, in the sense that the correlation of γ with the spacecraft trajectory is negligible. On the other hand, for a ranging GR test the limiting systematic error, after calibration for plasma, is the trajectory error. Fortunately for the Viking Lander experiment, the orbit of Mars is well known, but for spacecraft, there is always a difficulty of separating the relativistic ranging delay and the trajectory parameters. The Cassini experiment is no exception. We have successfully calibrated the Cassini X-band (7,175 MHz uplink, 8,425 MHz downlink) ranging data with a steady-state plasma model, in the sense that after fitting the calibrated data, the residuals are random with a standard error of 7

m. However, when we use only the ranging data to solve for the trajectory parameters and γ, the result is comparable to a previous spacecraft test with Voyager 2, $\gamma = 1 \pm 0.03$ [8]. Further, the ranging data add nothing to the Doppler determination of γ. The trajectory is improved by the ranging data, but because of the independence of γ and the trajectory parameters, the ranging data add no information to the γ determination.

Here we present GR results for an independent analysis of data from the Navigation System at JPL. A separate analysis, using data from JPL's Radio Science System, has been reported elsewhere [9]. The data processing in the two systems is quite different. The Radio Science System captures data, the open-loop (OL) data, by recording the downlink carrier signals at the DSN stations. The signals are digitally processed at JPL by means of software that constructs a phase record, or cycle count, as a function of time. The Navigation System uses separate receivers (Block V) to capture data, the closed-loop (CL) data, at the stations, but it is processed by hardware, in particular a cycle counter, and sent to JPL over a high-speed communication line. The main advantage of the OL data is that it can be sampled at a high rate. However, for dynamics experiments, including the Cassini solar conjunction experiment, high-rate data is not the issue. In fact we convert the raw CL data, available at one-second interval, to a frequency record at a 300 s interval. The cycle count is simply differenced at a 300 s interval and the result is divided by 300 s. We use $T_C = 300$ s because it is a good compromise between too small a sample interval, where high-frequency receiver noise can be a problem, and too long a sample interval, where too much data is lost for lack of enough continuous phase records over time T_C.

There is no real advantage to the OL data, except for one crucial capability. The OL data is recorded in three frequency channels, while the CL data, limited because only two CL receivers have been installed at DSS25 so far, can be recorded in only two channels. The Cassini radio system provides highly-accurate measurements of the GR frequency shift basically because it is a three-link system [7, 10]. The first two links, provided by the Cassini transponder are an X-band up, X-band down link, also used for the ranging data mentioned above, and the same X-band up, but Ka-band down (32,028 MHz). The third link, Ka-band up (34,316 MHz), Ka-band down (32,028 MHz), is provided by a spacecraft Ka-band frequency translator (KaT) and associated electronic components developed by the Italian Space Agency. The Cassini CL data are available only at X-band up, X-band down and Ka-band up, Ka-band down. While the three-link OL data provide an almost complete removal of the plasma noise [11], if not too close to the solar limb, the two-link CL data provide only a partial removal. For this reason, we use three-link OL plasma calibrations available from the Cassini Radio Science archive, rather than an independent CL calibration with the Cassini Navigation data. Similarly, calibrations provided by DSN water-vapor radiometry are available for the wet component of Earth's troposphere. However, the application of these calibrations is time consuming and difficult. They are not applied here. All other calibrations available to the Navigation System are applied, including the dry component of the troposphere, the Earth's polar motion, and variations in the Earth's rotation rate.

The fit to the CL Doppler data is accomplished by means of JPL's institutional navigation software, the Orbit Determination Program (ODP). This software system provides the most complete and sophisticated model available for fitting DSN data from deep space missions. In fact the ODP model, when applied to the Cassini mission, is so complete that very few parameters are required to fit the calibrated Doppler data to the noise level. We include eight parameters in the weighted-least-squares solution: first, the six initial conditions (state) for the spacecraft trajectory; secondly, a constant radial acceleration a_r, primarily to account for the spacecraft's thermal emission, but also for smaller effects such as unmodeled solar-pressure, beamed radio emission, and a possible contribution from the Pioneer anomaly [12]; and finally the relativity parameter γ. The inclusion of one or more parameters in the model for solar radiation pressure acting on the spacecraft is unnecessary. The solar-pressure model for the Cassini spacecraft is well determined from data taken many months before the 2002 solar conjunction, and at distances closer to the Sun. The thermal emission is more problematical, and it is appropriate to include a constant acceleration in order to account for thermal emission over the 27 days of the conjunction experiment.

The solution for the two parameters, exclusive of the six state parameters, is:

$$\begin{aligned} \gamma - 1 &= (-4.8 \pm 5.7) \times 10^{-5} \\ a_r &= (-26.7 \pm 1.1) \times 10^{-8}\ \mathrm{cm/s^2} \end{aligned} \tag{5}$$

where the uncertainties include both random and systematic error, with each of the 1569 Doppler measurements weighted according to the rms residual in its group of measurements on a pass by pass basis. The plot of the residuals (observed Doppler minus modeled Doppler) is displayed in Fig. 1, where the residual in Hz has been converted to Doppler velocity v_r in

μm/s according to,

$$v_r = \frac{1}{2} c \frac{\Delta\nu}{\nu} \tag{6}$$

The plasma calibrations are applied to the data on the X-band up and X-band down link, hence the frequency ν in Eq. 6 is 8,425 MHz, and the factor of 1/2, consistent with Eq. 4, accounts for the fact that the Doppler shift is measured for a two-way radio carrier wave. The plot in Fig. 1 indicates that there are 17 Doppler passes over the 27-day experiment. We have eliminated a pass of Doppler closest to conjunction on egress because of excessive plasma noise, even after calibration. This is consistent with an earlier analysis which concluded that because of multipath effects, Doppler data taken too near the solar limb are useless [11]. Similarly, the first two passes after conjunction in Fig. 1 contain fewer points than other passes. There is some justification to delete these two passes as well, but the plasma outliers are obvious, and the remaining points kept in the fit cluster about the zero line. There is a clear indication that the plasma calibration works as well for these few points as it does for all the remaining passes farther from the solar limb. There is no justification to further edit the 1569 residuals shown in Fig. 1.

Some discussion of error is in order. The ODP computes both random and systematic error based on our weighting by the rms residual on a pass by pass basis. Indeed a histogram for the residuals of Fig. 1 demonstrates that these residuals are indeed random and normally distributed. Therefore, on average, the accuracy of a single measurement of γ is $5.7\sqrt{1569} = 226$ in units of 10^{-5}, although the accuracy varies from a minimum of about 50×10^{-5} close to conjunction to a maximum of about 350×10^{-5} far from conjunction. The result of Eq. 5 is obtained by digging into the random Doppler noise, dominated by the wet component of the Earth's troposphere, by a factor of about eight for points closest to the solar limb. As for systematic error, the ODP includes this automatically by means of model parameters that contribute to the error, but are not included in the solution. This assures that the error does not fall below a lower limit established by those parameters, including errors in solar pressure, station locations, dry-troposphere, polar motion, and solid-Earth tides. In addition, systematic error introduced by unknown non-gravitational spacecraft forces is included in the parameter a_r. The spacecraft state is included in the set of parameters solved for, but the resulting error in the location of the ray path with respect to the solar limb is far too small to have any effect on the error in γ. The trajectory determination essentially establishes the Doppler baseline (mean) for the independent determination of

γ. Thrust forces introduced by the attitude control system can be ignored. The Cassini spacecraft was under the attitude control of reaction wheels during the experiment, and hence was not jetting gas. However, without the reaction wheels, the experiment would have been impossible, and in fact, our plan to repeat the experiment during the June - July 2003 conjunction was foiled because of a problem with the wheels. This 2002 conjunction is all we have, but it is good enough. A repeat of the experiment in 2003, without flaws, would have undoubtedly improved the γ result by more than the square root of two, but probably by no more than a factor of two.

Finally, the error in a_r from 27 days of Cassini Doppler data is about two times better than the result from 11 years of Pioneer 10 Doppler data [12]. However, unlike Pioneer, the result is not anomalous. Both Pioneer and Cassini are powered by radioisotope thermoelectric generators (RTG), but on Pioneer they are mounted on booms and radiate the bulk of their thermal output isotropically into space without reaching the spacecraft. On the other hand for Cassini, the RTG's are mounted on the spacecraft bus beneath the high-gain parabolic dish antenna. Their thermal output is controlled by reflection and absorption by the antenna and other spacecraft parts. It is difficult to model, although it should be directed toward the Earth, as confirmed by the negative sign in the solution for a_r. However, the uncertainty in the thermal model overwhelms any plausible application of the Pioneer anomaly to Cassini.

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[1] O. I. Yakolev *Space Radio Science* (Taylor and Francis, New York, 2002), Earth Space Institute Book Series 8.

[2] See the discovery paper by I. I. Shapiro, Phys. Rev. Lett. **13**, 789 (1964). The derivation of the effect and additional references can be found in C. M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, England, 1993), Rev. ed.

[3] R. D. Reasenberg, I. I. Shapiro, P. E. MacNeil, R. B. Goldstein, J. C. Breidenthal, J. P. Brenkle, D. L. Cain, T. M. Kaufman, T. A. Komarek, and A. I. Zygielbaum, Astrophys. J. **234**, L219 (1979).

[4] T. D. Moyer, Celestial Mech. **23**, 33 and 57 (1981).

[5] I. Ciufolini and J. A. Wheeler, *Gravitation and Inertia* (Princeton University Press, Princeton, 1995).

[6] B. Bertotti and G. Giampieri, Class. Quantum Grav. **9** 777 (1992).

[7] L. Iess, G. Giampieri, J. D. Anderson, and B. Bertotti, Class. Quantum Grav. **16**, 1487 (1999).

[8] T. P. Krisher, J. D. Anderson, and A. H. Taylor, Astrophys. J. **373**, 665 (1991).

[9] B. Bertotti, L. Iess, P. Tortora, and J. D. Anderson, Nature, accepted (2003).

[10] B. Bertotti, G. Comoretto, and L. Iess, Astron. Astrophys. **269**, 608 (1993).

[11] B. Bertotti and G. Giampieri, Solar Physics **178**, 523 (1998).

[12] J. D. Anderson, P. A. Laing, E. L. Lau, A. Liu, M. M. Nieto, and S. Turyshev, Phys. Rev. D**65**, 082004 (2002).

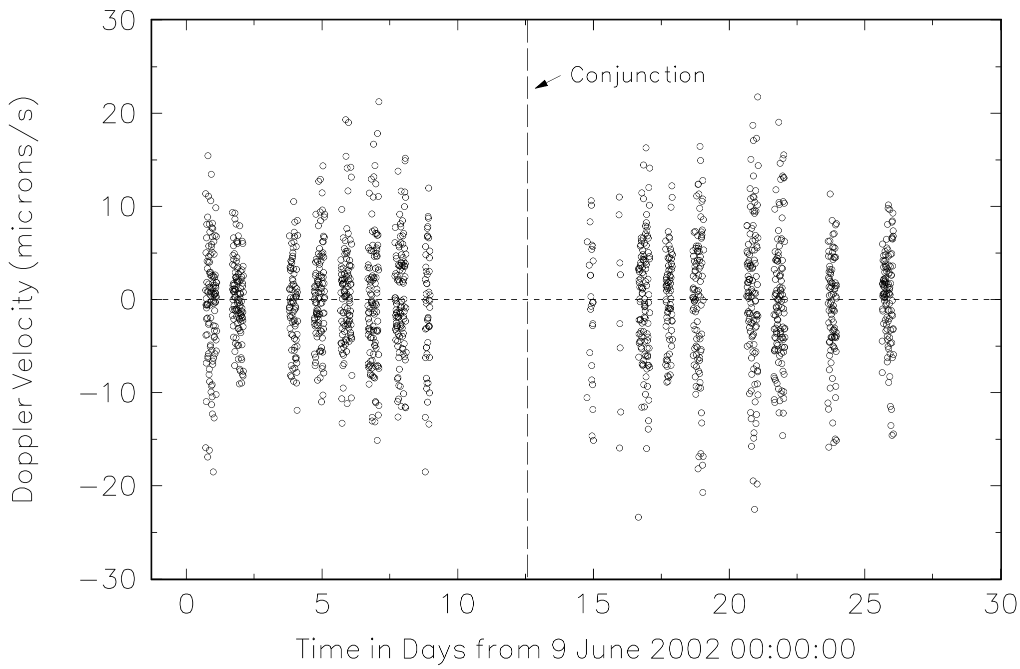

FIG. 1: Doppler velocity residuals (μm/s) after fitting for the trajectory model, a constant non-gravitational acceleration, and the relativity parameter γ. There are 1569 residuals, normally distributed about a zero mean, with standard deviation 6.25 μm/s.