The Speed and Lifetime of Cosmic Ray Muons

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We measure the mean lifetime and speed of cosmic ray muons at sea level using coincidence techniques with scintillation detectors. We find the mean speed v of muons traveling near normal to earth's surface to be 29.8 ± 2.5 cm/ns, corresponding to $\beta_\mu = 0.994 \pm 0.08$ and γ_μ of 9.14. Our result is in agreement with the accepted average speed of v_μ, which is listed as between 0.994 or 0.998 depending on the source [6]. Additionally, we find the mean lifetime of cosmic ray muons to be ultimately 2.19 ± 0.01 μs, in agreement with prior experiments that have found the value to be 2.197 μs [2]. We find that classical physics is unable to explain our results and that the travel of muons through the Earth's atmosphere can only be sufficiently understood within the framework of Special Relativity.

1. INTRODUCTION

Muons (μ^-) and antimuons (μ^+) created in the upper atmosphere ($\sim 15km$ [4]) are secondary products of interactions between highly energetic cosmic rays and the nuclei of atmospheric particles. They are the direct result of decay of two different species of pions, π^- and π^+. Since they interact weakly and have a mass much greater than that of the electron, muons are highly penetrating particles able to reach the ground. The "cascade" of muons detected at sea level is directionally dependent on the zenith angle, θ, as $\cos^2(\theta)$. It is therefore much more likely to detect muons traveling normal to the earth's surface than parallel.

2. EXPERIMENTAL GOALS - THEORY

Ultimately, we aim to compare the travel time of muons through the atmosphere with the mean lifetime of the muon. We determine both values experimentally, and carry out computations to find the maximum expected flux of muons at sea level according to two theories of kinematics: classical theory of physics and Special Relativity. The fundamental distinction between these two theories are their sets of frame-invariant parameters. In classical physics, time is an invariant parameter, meaning, that the time measured in one frame is equal to that measured in any other frame. This assumption breaks down in Special Relativity, in which an effect called "time dilation" in essence "slows down" time in a moving frame with respect to a frame at rest. The derived result is that cosmic ray muons propagating toward earth at a significant fraction of the speed of light experience an elapsed time that is less than the elapsed time on earth by the Lorentz factor γ, such that,

$$t_\mu = t_{earth}/\gamma \tag{1}$$

where $\gamma = \frac{1}{\sqrt{1-(v_\mu/c)^2}}$ [7]. In Special Relativity, the upper limit on the relative speeds between inertial frames is c, the speed of light, at 29.98 cm/ns. As the velocity approaches c, γ grows, and time dilation becomes significant enough to be measured. With knowledge about the muon's travel time (measured in the earth frame) and mean lifetime, we can calculate the muon flux at sea level predicted by the two theories. Finally, a comparison with actual flux measurements will promote one theory above the other.

3. THE MEAN SPEED OF MUONS

3.1. Theory and Method

We determine the mean speed of muons by using two flat, broad scintillator paddles separated by a vertical adjustable height, D. By placing coincidence requirements on the signals from these detectors, we restrict our data to only those particles (muons) that have passed through both detectors, and find the time lag between the top and bottom events, T. We can then calculate the muon mean speed through the relationship,

$$\Delta D = v_\mu \Delta T \tag{2}$$

Rather than relying on our electronics to determine time delays on the order of nanoseconds, we can easily obtain greater accuracy in our data and eliminate many puzzling systematics by measuring the *difference* in travel time of muons, T_1 and T_2, between the paddles separated by two unique distances, D_1 and D_2, as opposed to the absolute transit time for a single separation. As long as our set up is unchanged between the two measurements, any offset in time, t_0, will drop out of the equation when the difference is taken.

3.2. Equipment and Calibration

Our experimental set-up for the determination of muon speed can be found in Figure 1. The length of cable for

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the delay of the second pulse can be chosen arbitrarily, given that the delay is undisturbed between sets of differential measurements.

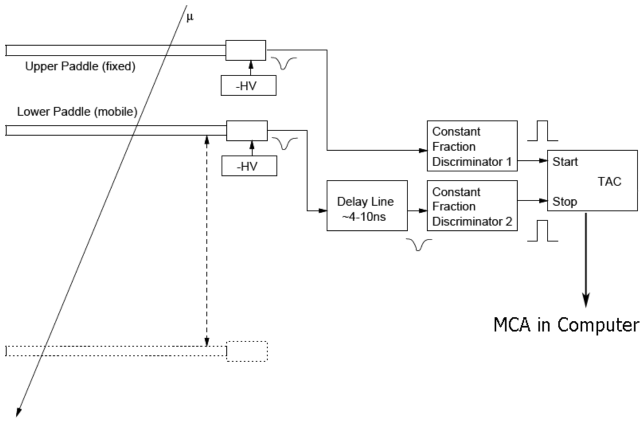

FIG. 1: Schematic of detector to MCA chain. Diagram modified from 8.13 Lab Guide [4]. The crucial component in our setup is the Time-to-Amplitude Converter (TAC), which takes start and stop pulses separated by time t and generates a single pulse with amplitude proportional to t. This can then be recognized by the MCA and sorted into 2048 bins based on amplitude of pulse received. The result is a frequency spectrum of time delays of pulses received from the upper and lower paddles.

We calibrate our setup by feeding it START and STOP pulses of known separations in time. This way, we can tune the resolution (ns per bin) in the MCA to the desired value for this experiment, 20 ns/bin. A calibration is taken before every set of distance trials to determine the channel number vs. time proportionality. Figure 2 shows a sample calibration.

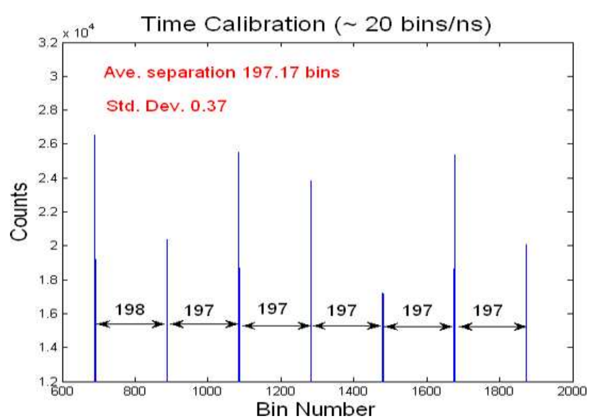

FIG. 2: Pulses from the Time Calibrator as read by the MCA. This calibration corresponds to a resolution of 19.7 bin/ns.

3.3. Slant Range

Muons impact our detectors from a whole distribution of directions defined by the spherical parameters θ and ϕ. The distance traveled by any single muon between the detectors will be larger than their separation, D if θ is non-zero. Using the angular distribution of incident muon flux at a given elevation [4],

$$I(\theta) \propto cos^2(\theta) \tag{3}$$

we run a Monte Carlo simulation to determine a mean slant range for any D. Incident muons generated each trial are assigned a position coordinate in the top paddle, $(x, y, z = D)$ and a direction of propagation that is uniformly random in ϕ (the azimuthal angle) and weighted in θ according to Equation 3.

Each simulated muon is allowed to propagate in a straight line. When the particle crosses the $z = 0$ plane (representative of the bottom paddle locaton), the particle is assessed for a "hit" or a "miss". For the "hits", the distance traveled is found by numerical integration.

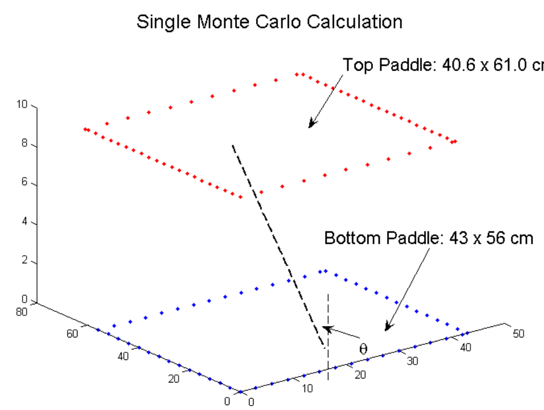

FIG. 3: Graphical representation of a single trial in the Monte Carlo simulation for a separation of 10cm.

The average values for slant ranges differ from the measured separation by a few centimeters at the nearest paddle positions and are nearly equal to the separation at the farthest, as one would expect.

3.4. Data and Analysis

Figure 4 gives the transit time distribution for muons measured with a small separation between the top and bottom paddles. Notice the skewedness of the spectrum toward longer time intervals between top and bottom incidence; we quantify this using the difference between the mean and the mode of the distribution. This effect is most dramatic when the detectors are close, and nearly indiscernible at larger separations. One may attempt to account for this shift using the effects of the slant range. However, a 5000 trial simulation using our Monte Carlo script tells us the expected shift due to this effect is small, at the most a difference of about 3 cm. The corresponding effect we observe in the data (translated into effective shift in separation) is a whole magnitude larger, on the order of 30 cm. Since the mean is now

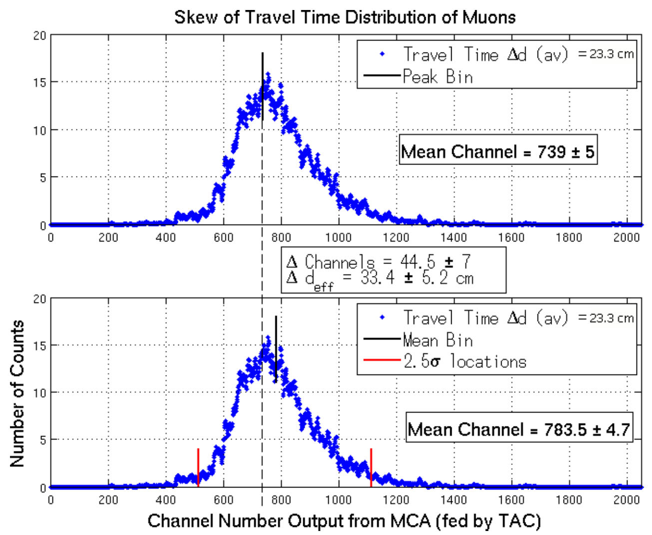

FIG. 4: Transit time distribution of incident muons with a detector separation of 23.3 cm. The two graphs show the same spectrum with the peak bin and mean bin marked. The peak bin was determined by inspection and mean found via a weighted average over the $\pm 2.5\sigma$ range for the peak. Their disparity is quantified in terms of effective difference in distance traveled.

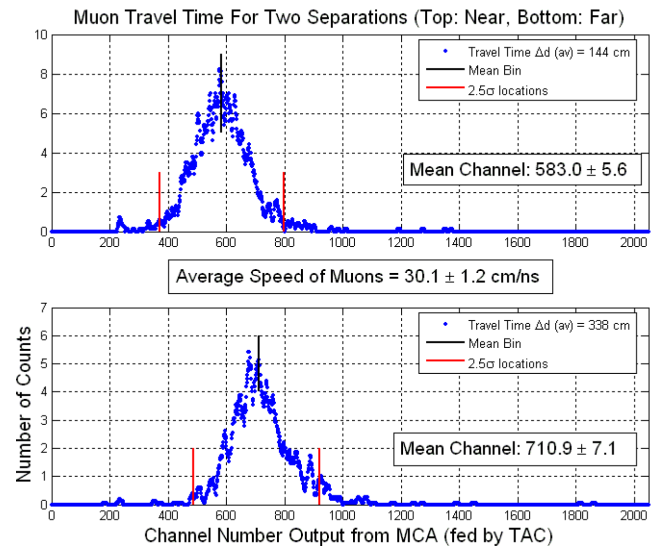

FIG. 5: Top: 144 cm separation, Bottom: 338 cm separation. Calibration is 19.7 bins/ns. Mean channel computed by taking weighted average within about 2.5σ of peak in either direction.

shifted toward higher transit times, and all separations are not uniformly affected, this results in a systematic effect which will not be eliminated by taking the difference between measurements.

We suspect that this effect may be a combination of a couple factors. First, it may be reflective of a change in the velocity profile of the intercepted beams of muons between the near and far peaks, since muons arriving from directions of large θ will tend to have reduced velocities due to additional travel through the atmosphere. It may additionally be caused by a change in response of the detector and photomultiplier to longer duration/brighter signals experienced by muons impacting at large angles θ with respect to normal and taking a longer path through the detectors. Additional forces may be at work to complicate the issue further.

In order to minimize an effect that we cannot fully quantify in our measurements of average muon speed, we decided to avoid coincidence measurements of muons arriving at large polar angles altogether. We restrict ourselves to muons from the near normal direction by taking measurements at large separations. Average slant distances are used in our calculations in lieu of measured separation. Our results are summarized in Figure 5.

The result obtained from this trial was an average speed of $30.1 \pm 1.2 cm/ns$. This is greater than the speed of light c. Have we really found a particle that travels faster than light? It is believed there are perhaps some systematics at play[1].

[1] We are grateful to Professor Becker for this recommendation

3.4.1. With Systematic Error Correction

The photomultiplier tubes (PMTs) for the top and bottom detectors have been found to behave slightly differently, and due to some malfunctioning parts, produce waveforms of different shape for pulses in the top and bottom detectors that vary with separation distance. This is a systematic error that can either reduce or increase the average measured time interval of muon travel. However, it need not to be characterized. If the original effect was a lengthening of the average time interval, by switching the START and STOP nodes on the TAC, we will now measure an interval shortened by the same amount, their average representative of a more accurate value free of this particular systematic error.

Again, the MCA was calibrated to a resolution of approximately 20 bins/ns. This time, the transit time for the larger separation appears on the lower end of the time scale. Our results, once again, are summarized in a graph (see Figure 6).

The average speed obtained in the reverse trials was 29.6 ± 2.2 cm/ns, slightly smaller than the speed of light. Thus, we obtain our result for the average speed of cosmic ray muons, v_μ:

$$v_\mu = 29.8 \pm 2.5 \; cm/ns \tag{4}$$

This value is 99.4% of c. We can see that the upper limit on the relative motion of inertial frames holds. Since we have determined the speed of the muon, we can easily calculate using Newtonian mechanics the time required for the muon to travel from a height of 15 km (where it is created) to sea level (where the lab is located). This time is approximately 50.3 μs for a muon particle in the frame of the lab. We are interested in comparing this necessary flight time with the mean life of a muon.

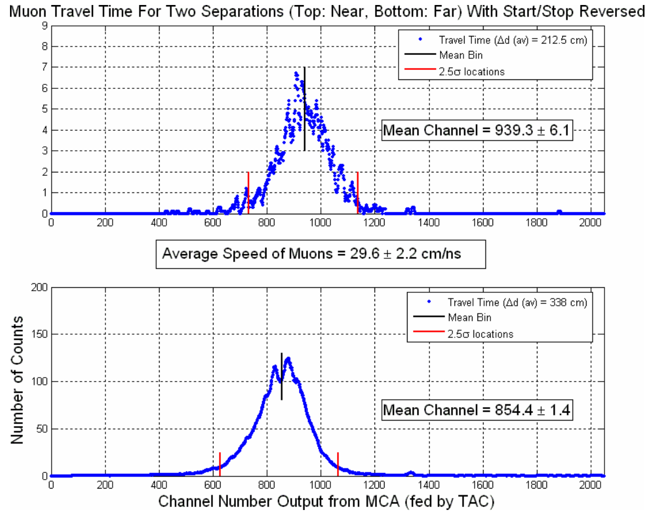

FIG. 6: Same measurements with START and STOP reversed. Top: 212.5 cm separation, Bottom: 338 cm separation. Calibration is 19.7 bins/ns. The bottom trial was allowed to integrate overnight.

4. THE MEAN LIFE OF MUONS

4.1. Theory and Set-up

Muons and antimuons decay through the weak interaction to produce electrons, positrons, and two species of neutrinos [3].

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \quad (5)$$
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \quad (6)$$

Like any decay mechanism, this process can be characterized by its mean lifetime, τ, representative of the amount of time that must elapse before the given population of particles is reduced by a factor of e. The governing equation for this process is,

$$I(t) = I_0 e^{-t/\tau} \quad (7)$$

When applied to cosmic ray muons, this equation can be used to determine the approximate expected flux of muons, $I(t)$, at sea level given the travel time t from its initial height of 15 km. If we make the (incomplete) assumption that the decrease in flux is solely due to muon decay, we can calculate an upper limit for the incident flux of muons at sea level.

We set up an experiment to find the mean lifetime, τ, of muons. By taking advantage of the nature of the decay function, we can set any arbitrary time as $t = 0$ and extract the same value τ from its subsequent decay probabilities. Our set-up rests on coincidence measurements in a 20.3 kg cylindrical plastic scintillator. A muon that comes to rest in the detector induces one signal upon entry and another upon decay. The time delay between the two pulses is plotted on an MCA display. Accidentals are coincidences due to the passing of two unrelated muons through the detector within the coincidence time limit. We restrict the TAC limit to 10 μs, a region in which the rate of accidentals is far lower than the rate of actual decay events. A schematic is provided in Figure 7.

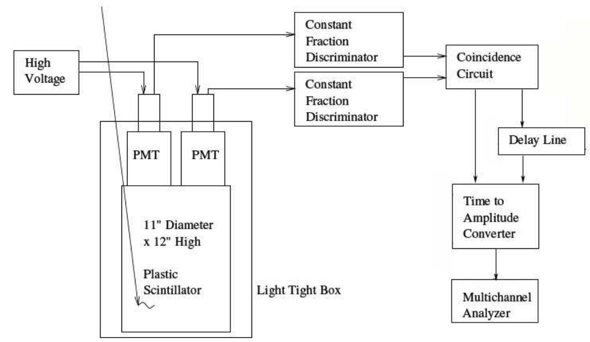

FIG. 7: Schematic modified from [4]. The full range of the TAC was set as 10 μs for 2048 channels of the MCA.

Since we require muons to come to rest among the atoms of the scintillator, another decay mechanism may be observable in addition to the isolated decay of the muon. Muons with a negative charge are susceptible to capture by positive high-Z nuclei of atoms, and are absorbed in the faster process [1],

$$\mu^- + p \rightarrow n + v \quad (8)$$

Although the effect is understood to be relatively small with low-Z material such as carbon (Z = 6), and negligible in the case of hydrogen (Z = 1) [1], this is an effect we should correct for in the hydrocarbon scintillator.

4.2. Data and Analysis

Raw data integrated for approximately 65,000s is displayed in Figure 8. We eliminated the displaced counts (as evidenced by the hole and the peak) from our data set and took running averages with a windowsize of 15 bins in order to remove any zero-bins from the high end of our spectrum which would affect our fit. This smoothing of our data reduces the error on each data point by a factor of about $\sqrt{15}$.

We fit our data in stages, pausing to check the plausibility of each result and record any changes in the value obtained for τ caused by a particular correction. We break our data down into three contributions which add linearly to produce our resultant curve.

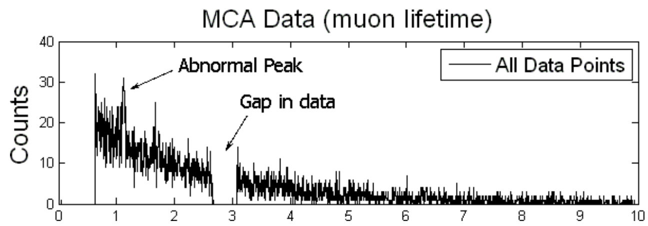

FIG. 8: Raw MCA data obtained in muon lifetime measurement. Computer hardware errors are likely behind the gap in data around 3 μs and the corresponding peak around 1 μs. These segments were ignored in our analysis.

4.2.1. Baseline Accidentals

Approximately 20 muons pass through the cylindrical scintillator per second[2]. Their arrival can be modeled by a Poisson distribution, with the probability of two poisson events occuring exactly time t apart obeying the relation,

$$P(t) = re^{-rt} \tag{9}$$

with $r = 20\ /s = 2 \times 10^{-5}/\mu s$. This is a very slowly decaying curve, with the difference in counts expected between the lowest and highest bin in the MCA on the order of 200 parts per million. Therefore, we will neglect the exponential nature of the spectrum of accidentals and treat the distribution as a constant. We compute the value of this constant to be 0.1338 /bin for our integration time of 65,000s and the median bin at $t = 5\ \mu s$.

4.2.2. Isolated Muon Decay

The desired result, of course, is the lifetime of the free muon, τ. Since cosmic rays produce both negative and positively charged muons, at a ratio of 44% to 56% [5], each species will have its own mean life-time, represented by τ_{μ^+} and τ_{μ^-}. In free space, these quantities are identical, and add with amplitude,

$$I_0 = I_{0,\mu^+} + I_{0,\mu^-} \tag{10}$$

where $I_{0,\mu^+} = 0.56 I_0$ and $I_{0,\mu^-} = 0.44 I_0$ so that determination of either quantity, τ_{μ^+} or τ_{μ^-} results in the determination of τ. This, however, is not the case within the plastic scintillator.

4.2.3. Negative Muon Capture

By the process described in Section 4.1, the average life of negative muons is reduced by a certain factor based on the probability of capture. The average life of positive muons is largely unchanged and continues to represent the rate of isolated muon decay. According to [1], the theoretical value for the mean lifetime of μ^- in carbon is approximately 1.7 μs (taken as the average of the two values obtained on page 170 by varying a constant, Z_0). We fit our data to the sum of both the positive and negative muon decay curves.

$$I(t) = I_0 \left(0.56 e^{-t/\tau} + 0.44 e^{-t/1.7}\right) + b \tag{11}$$

where t is in μs, and b is the baseline count per bin.

4.3. Results of Curve Fits with Systematic Corrections

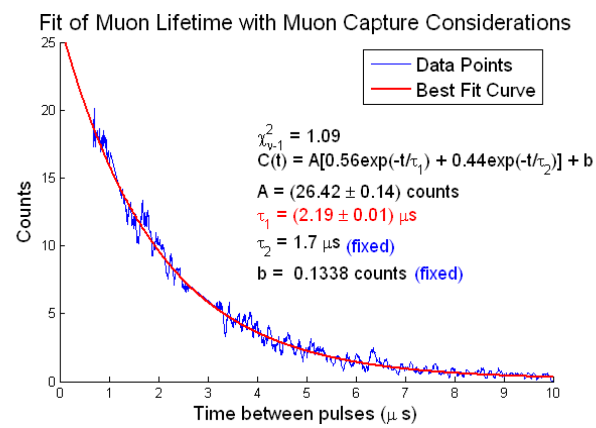

FIG. 9: Curve fit determination of muon mean lifetime, τ, with consideration of background accidentals and μ^- capture.

Method	Lifetime (μ sec)	Mass (MeV/c^2)
1 exp	1.97 ± 0.01	107.89 ± 0.11
1 exp, fixed bg	2.00 ± 0.01	107.57 ± 0.11
2 exp, fixed bg	2.19 ± 0.01	105.63 ± 0.10
NIST values	2.19703 ± 0.00004	105.65839 ± 0.000034

TABLE I: Results for mean lifetime and calculated rest mass of muons determined from sequential error-correcting fits compared with values from NIST physics database.

Table I gives our summarized results as each of our identified systematic corrections are applied. The first fit is simply to an exponential of the form $Ae^{-t/\tau} + b$ as recommended by the lab guide. Our result is $\tau = 1.97\ \mu s$. In the next fit, the baseline b is fixed as 0.1338 counts/bin, the value determined in Section 4.2.1. τ increases slightly to 2.00 μs. Finally, in addition to the previous correction, we separate out the effect of muon capture. Our final result for the mean lifetime of the muon is,

$$\tau = 2.19 \pm 0.01 \mu s \tag{12}$$

[2] This was verified using a pulse counter

in agreement with the accepted value of 2.197 μs [2]. Figure 9 shows the final curve fit with all corrections applied. We find the rest mass of the muon using the relation [4],

$$(m_\mu c^2)^5 = \frac{192\pi^3\hbar}{G_F^2\tau} \quad (13)$$

$$\Rightarrow m_\mu c^2 = 105.63 \pm 0.10 MeV \quad (14)$$

5. BRIEF DISCUSSION OF ERROR

We have determined the speed and mean lifetime of cosmic ray muons within errors of approximately 10% and 1%, respectively. By simply increasing our integration time, we can improve the precision of our determinations of both quantities. The impact will be greatest on our peak measurements in Section 3.4. If we double our integration time, we can reduce by a factor of $\sqrt{2}$ the error on our mean channel, and correspondingly the error on speed of muons decreases. We were limited, in our case, by the staggered schedule of overnight integration shared by three other lab groups, however, this would be the most natural first step in improving our measurements in the future.

Systematic corrections were generally used in dealing with equipment errors. However, one source of obvious but unpredictable error can be spotted in the MCA data collected for various components of this experiment. It is most evident in our overnight lifetime data, however, it may have also have manifested itself to a lesser extent during our investigation of muon speed. In the bottom graph in Figure 6, we can see a valley where the maximum counts are expected and a peak feature many standard deviations from the center of the distribution. It's difficult to quantify the effect of these irregularities, especially in the data sets in which they are less distinct, however, due to its central location, we trust that it did not play too dramatic a role in our determination of mean channel.

6. CONCLUSIONS: RELATING SPEED WITH MEAN LIFE

We use our results for the speed of the muon, $v_\mu = 29.8 cm/ns$, and its mean life time, $\tau = 2.19 \mu s$, to calculate expected flux of particles at sea level. Equation 7 tells us that according to classical mechanics, the elapsed time between the creation of the muon and its detection on earth is 50.3 μs. We should see a flux of mere 6×10^{-12} $cm^{-2}\ s^{-1}\ sr^{-1}$: a 10 orders of magnitude reduction from the initial flux at 15 km. But we clearly observed a much larger flux than this.

If we instead turn to the theory of Special Relativity, the situation can be seen from two perspectives: from the rest frame of the muon and from the rest frame of the earth. In Section 2, we've already found the relationship between the time experienced by the muon and the time measured on earth. Using that relation, and the measured γ of 9.14, we find that,

$$t_\mu = t_{earth}/\gamma = 5.5\ \mu s \quad (15)$$

The resultant sea-level flux of muons is reduced by a factor of 10 from its peak intensity. In fact, on earth, we measure a beam of muons reduced in intensity by only one order of magnitude, $I_{obs} = 0.83 \times 10^{-2}$ $cm^{-2}\ s^{-1}\ sr^{-1}$. So the results are in agreement.

Alternatively, one can envision the muon at rest and the earth in motion toward it. In the rest frame of the muon, the earth is moving at $\beta = 0.994$, and its atmosphere is contracted by the Lorentz factor, $\gamma = 9.14$ with respect its rest length of 15 km.

$$L_\mu = L_{earth}/\gamma = 1641\ m \quad (16)$$

At a speed of 29.8 cm/ns, the muon can then traverse that distance in 5.5 μs. We've arrived at the same result. At the conclusion of this experiment, we have found strong evidence in support of the theory of Special Relativity and must appreciate its power to provide solutions and explanations where Classical Mechanics cannot.

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Acknowledgments

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All non-linear fits were made with the MATLAB scripts made available to us on the Junior Lab website: http://web.mit.edu/8.13/www/jlmatlab.shtml.