ART. XXXVI.—*Influence of Motion of the Medium on the Velocity of Light;* by ALBERT A. MICHELSON and EDWARD W. MORLEY.†

THE only work of any consequence, on the influence upon the velocity of light of the motion of the medium through which it passes, is the experiment of Fizeau. He announced the remarkable result that the increment of velocity which the light experienced was not equal to the velocity of the medium, but was a fraction x of this velocity which depended on the index of refraction of the medium. This result was previously obtained theoretically by Fresnel, but most satisfactorily demonstrated by Eisenlohr,‡ as follows:
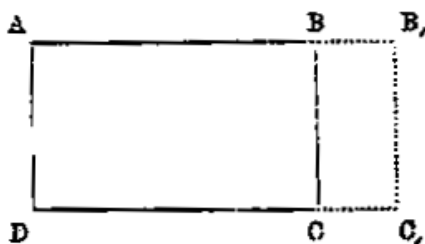
Consider the prism AC in motion relatively to the ether in direction AB with velocity θ. Suppose the density of the external ether to be 1 and of the ether within the prism, $1+\Delta$. In the time dt the prism will advance a distance θdt=BB, At

* J. pr. Ch., 1·319 (Rammelsberg, Mineral Chemie, 185).
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‡ Verdet. Conferences de Physique, ii, 687.

the beginning of this time the quantity of ether in the volume BC, (if S=surface of the base of the prism,) is $S\theta dt$. At the end of the time the quantity will be $S\theta dt(1+\varDelta)$. Hence in this time a quantity of ether has been introduced into this volume equal to $S\theta dt\varDelta$.

It is required to find what must be the velocity of the ether contained in the prism to give the same result. Let this velocity be $x\theta$. The quantity of ether (density$=1+\varDelta$) introduced will then be $Sx\theta dt(1+\varDelta)$ and this is to be the same as $S\theta dt\varDelta$, whence $x=\dfrac{\varDelta}{1+\varDelta}$. But the ratio of the velocity of light in the external ether to that within the prism is n, the index of refraction, and is equal to the inverse ratio of the square root of the densities, or $n=\sqrt{1+\varDelta}$ whence $x=\dfrac{n^2-1}{n^2}$ which is Fresnel's formula.*

* The following reasoning leads to nearly the same result; and though incomplete, may not be without interest, as it also gives a very simple explanation of the constancy of the specific refraction.

Let l be the mean distance light travels between two successive encounters with a molecule; then l is also the "mean free path" of the molecule. The time occupied in traversing this path is $t=\dfrac{a}{v_{/}}+\dfrac{b}{v}$, where a is the diameter of a molecule, and $b=l-a$, and $v_{/}$ is the velocity of light within the molecule, and v, the velocity in the free ether; or if $\mu=\dfrac{v}{v_{/}}$ then $t=\dfrac{\mu a+b}{v}$. In the ether the time would be $t_{/}=\dfrac{a+b}{v}$, hence $n=\dfrac{t}{t_{/}}=\dfrac{\mu a+b}{a+b}$.   (1)

If now the ether remains fixed while the molecules are in motion, the mean distance traversed between encounters will no longer be $a+b$, but $a+a+b+\beta$; where a is the distance the first molecule moves while light is passing through it, and β is the distance the second one moves while light is moving between the two. If θ is the common velocity of the molecules then $d=\dfrac{\theta}{v_{/}}a$, and $\beta=\dfrac{\theta}{v-\theta}b$. The time occupied is therefore $\dfrac{a}{v_{/}}+\dfrac{b}{v-\theta}$ or $\dfrac{\mu a}{v}+\dfrac{b}{v-\theta}$. The distance traversed in this time is $a+b+\left(\dfrac{\mu a}{v}+\dfrac{b}{v-\theta}\right)\theta$; therefore the resulting velocity $v=\dfrac{a+b}{\dfrac{\mu a}{v}+\dfrac{b}{v-\theta}}+\theta$.

Substituting the value of $n=\dfrac{\mu a+b}{a+b}$ and neglecting the higher powers of $\dfrac{\theta}{v}$, this becomes $v=\dfrac{v}{n}+\left(1-\dfrac{1}{n^2}\dfrac{b}{a+b}\right)\theta$.   (2)

But $\dfrac{v}{n}$ is the velocity of light in the stationary medium; the coefficient of θ is therefore the factor $x=\dfrac{n^2-1}{n^2}+\dfrac{1}{n^2}\dfrac{a}{a+b}$.   (3)

It seems probable that this expression is more exact than Fresnel's; for when the particles of the moving medium are in actual contact, then the light must be accelerated by the full value of θ; that is the factor must be 1, whereas $\dfrac{n^2-1}{n^2}$ can

Fresnel's statement amounts then to saying that the ether within a moving body remains stationary with the exception of the portions which are condensed around the particles. If this condensed atmosphere be insisted upon, every particle with its atmosphere may be regarded as a single body, and then the statement is, simply, that the ether is entirely unaffected by the motion of the matter which it permeates.

It will be recalled that Fizeau[*] divided a pencil of light, issuing from a slit placed in the focus of a lens, into two parallel beams. These passed through two parallel tubes and then fell upon a second lens and were re-united at its focus where they fell upon a plane mirror. Here the rays crossed and were returned each through the other tube, and would again be brought to a focus by the first lens, on the slit, but for a plane parallel glass which reflected part of the light to a point where it could be examined by a lens.

At this point vertical interference fringes would be formed, the bright central fringe corresponding to equal paths. If now the medium is put in motion in opposite directions in the two tubes, and the velocity of light is affected by this motion, the two pencils will be affected in opposite ways, one being retarded and the other accelerated; hence the central fringe would be displaced and a simple calculation would show whether this displacement corresponds with the acceleration required by theory or not.

Notwithstanding the ingenuity displayed in this remarkable contrivance, which is apparently so admirably adopted for eliminating accidental displacement of the fringes by extraneous causes, there seems to be a general doubt concerning the results obtained, or at any rate the interpretation of these results given by Fizeau.

never be 1. The above expression, however gives this result when the particles are in contact—for then $b=0$ and $x=\dfrac{n^2-1}{n^2}+\dfrac{1}{n^2}=1$.

Resuming equation (1) and putting $a+b=l$ we find $(n-1)l=(\mu-1)a$. But for the same substance μ and a are probably constant or nearly so; hence $(n-1)l$ is constant.

But Clausius has shown that $l=\kappa\dfrac{\sigma}{\rho}a$, where κ is a constant, σ the density of the molecule; ρ, that of the substance; and a, the diameter of the "sphere of action." σ and a are probably nearly constant, hence we have finally $\dfrac{n-1}{\rho}=$ constant.

Curiously enough. there seems to be a tendency towards constancy in the product $(n-1)l$ for *different* substances. In the case of 25 gases and vapors whose index of refraction and "free path" are both known. the average difference from the mean value of $(n-1)l$ was less than 30 per cent. though the factors varied in the proportion of one to thirteen; and if from this list the last nine vapors (about which there is some uncertainty) are excluded, the average difference is reduced to 10 per cent.

[*] Ann. de Ch. et de Ph., III. lvii. p. 385, 1859.

This, together with the fundamental importance of the work must be our excuse for its repetition. It may be mentioned that we have tried to obtain formulated objections to these experiments but without success. The following are the only points which have occurred to us as being susceptible of improvement.

1st. The elimination of accidental displacement of the fringes by deformation of the glass ends of the tubes, or unsymmetrical variations of density of the liquid, etc., depends on the assumption that the two pencils have traveled over identical (not merely equivalent) paths. That this is not the case was proven by experiment; for when a piece of plate glass was placed in front of one of the pencils and slightly inclined, the fringes were displaced.

2d. The arrangement for producing the motion of the medium necessitated very rapid observation—for the maximum velocity lasted but an instant.

3d. The tubes being of necessity of small diameter and only their central portion being available (since the velocity diminishes rapidly toward the walls) involved considerable loss of light—which, having to pass through a slit was already faint.

4th. The maximum velocity (in the center of the tube) should be found in terms of the mean velocity. (Fizeau confessedly but guesses at this ratio.)

These are the suggestions which determined the form of apparatus adopted, a description of which follows:

The Refractometer.—After a number of trials, the following form was devised and proved very satisfactory. Light from a source at a (fig. 5) falls on a half silvered surface b, where it divides; one part following the path $bcdefbg$ and the other the path $bfedcbg$. This arrangement has the following advantages: 1st, it permits the use of an extended source of light, as a gas flame; 2d, it allows any distance between the tubes which may be desired; 3d, it was tried by a preliminary experiment, by placing an inclined plate of glass at h. The only effect was either to alter the width of the fringes, or to alter their inclination: *but in no case was the center of the central white fringe affected.* Even holding a lighted match in the path had no effect on this point.

The tubes containing the fluid were of brass, 28ᵐᵐ internal diameter; and, in the first series of experiments, a little over 3 meters in length, and in the second series, a little more than 6 meters. The ends of these tubes were closed with plane parallel plates of glass which were not exactly at right angles but slightly inclined so as to reflect the light below the telescope, which would otherwise be superposed on that which passed through the tubes. The tubes were mounted on a

wooden support entirely disconnected from the refractometer which was mounted on brick piers.
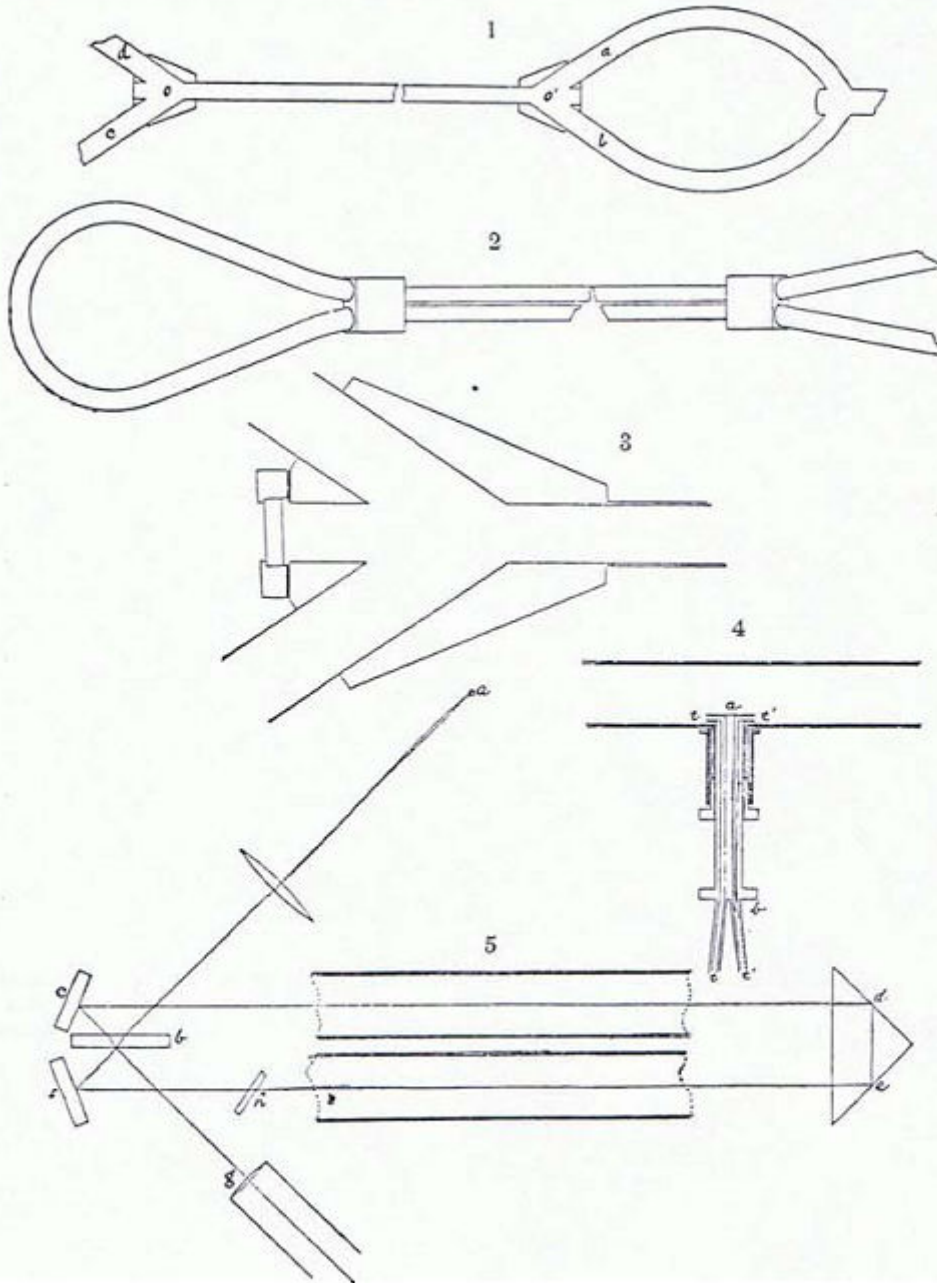
EXPLANATION OF FIGURES.

FIG. 1.—Vertical section through tubes. FIG. 2.—Plan of tubes. FIG. 3.—One end of tubes, showing glass plate inclined to axis. FIG. 4.—Gauge for velocity at different points. FIG. 5.—Plan of refractometer.

The flow of water was obtained by filling a tank four feet in diameter and three feet high, placed in the attic, about 25 meters above the apparatus, with which it was connected by a three inch pipe. The latter branched into two parts, and each

branch again into two; the two pairs being joined each to one of the tubes. The branches were provided with large valves, by turning which the current was made to flow in either direction through the tubes and into a large tank, from which it was afterward pumped up to the upper tank again. The flow lasted about three minutes, which gave time for a number of observations, with the flow in alternating directions.

Method of observation.—In the first series of observations a single wire micrometer was used in the eyepiece of the observing telescope, but afterward a double wire micrometer was employed. The tubes being filled with distilled water, the light from an electric lamp was directed toward the central glass of the refractometer and the latter adjusted by screws till the light passed centrally down both tubes, and then the right angled prism at the further end adjusted till the light returned and was reflected into the telescope, where generally two images were observed. These were made to coincide, and the fringes at once appeared. They could then be altered in width or direction by the screws, till the best result was obtained. A slight motion of one of the mirrors produced an inclination of the fringes, and the horizontal wire of the micrometer was placed at the *portion of the fringes which remained fixed,* notwithstanding the movement of the mirror. This adjustment was frequently verified, and as long as it was true, no motion of the tubes or distortion of the glasses could have any effect on the measurements. During this adjustment it was found convenient to have a slow current of water, to avoid distortions on account of unequal density.

The signal being given the current was turned on, and the micrometer lines set, one on each of the two dark bands on either side of the central bright fringe; and the readings noted. The difference between them gave the width of the fringe, and their mean, the position of the center of the central white fringe. This being verified the signal was given to reverse the current; when the fringes were displaced, and the same measurements taken; and this was continued till the water was all out of the upper tank. Following is a specimen of one such set of observations.

No. 63.

Direction of current.	+		—	
Micrometer wire.	*l.*	*r.*	*l.*	*r.*
	11	34	80	93
	13	35	71	88
	10	40	73	90
	13	38	67	92
	14	40	65	89
	10	35	61	94
Means...................	11·8	37·0	69·5	91·0

Width of fringe...........	48·6	60·5
Mean width..............		54·6 + (3·0 = index error)
Displacement	57·7	46·0
Mean displacement		51·8

$$\varDelta = \frac{51\cdot8}{57\cdot6} = \cdot899.$$

(Long tube, vertical fringes, full current.)

Velocity of water.—The velocity of the water in the tubes was found by noting the time required to fill a measured volume in the tank, and multiplying by the ratio of areas of tank and tube. This gave the mean velocity. In order to find from this the maximum velocity in the axis of the tube the curve of velocities for different radii had to be determined. This was done as follows: a tight fitting piston *ab* (fig. 4) containing two small tubes *tt*, *t*, *t*, was introduced into the tube containing the water. The ends of the tubes were bent at right angles in opposite ways, so that when the water was in motion the pressure would be greater in one than in the other. The other ends of the small tubes were connected to a U tube containing mercury, the difference in level of which measured the pressure. The pressures were transformed into velocities by measuring the velocity corresponding to a number of pressures. Following is the table of results :—

Pressures.	Velocities.	$\dfrac{v}{\sqrt{p}}$
26	393	77·1
108	804	77·1
190	1060	76·9
240	1190	76·8

It is seen from the approximate constancy of the last column that within limits of error of reading, the square roots of the readings of the pressure gauge are proportional to the velocities.

To find the curve of velocities along a diameter of the tube, the piston was moved through measured distances, and the corresponding pressures noted. The diameter of the tube was about 28^{mm}, while that of the small tubes of the gauge was but 2^{mm}, so that the disturbance of the velocity by these latter was small except very close to the walls of the tube. The portion of the piston which projected into the tube was made as thin as possible, but its effect was quite noticeable in altering the symmetry of the curve.

In all, five sets of observations were taken, each with a different current. These being reduced to a common velocity all gave very concordant results, the mean being as follows: *x*=distance from the axis in terms of radius; *v*=corresponding velocity in terms of the maximum.

$x.$	$v.$
0·00	1·000
·20	·993
·40	·974
·60	·929
·80	·847
·90	·761
·95	·671
1·00	·000

The curve constructed with these numbers coincides almost perfectly with the curve

$$v = (1 - x^2)^{.165.}$$

The total flow is therefore $2\pi \int_0^1 (1 - x^2)^{.165} x\, dx = \dfrac{\pi}{1·165}$. The area of the tube being π, the mean velocity $= \frac{1}{1·165}$ of the maximum; or the maximum velocity is 1·165 times the mean. This, then, is the number by which the velocity, found by timing the flow, must be multiplied to give the actual velocity in the axis of the tube.

Formula.

Let l be the length of the part of the liquid column which is in motion,

$u =$ velocity of light in the stationary liquid.

$v =$ velocity of light in vacuo.

$\theta =$ velocity of the liquid in the axis of tube.

$\theta x =$ acceleration of the light.

The difference in the time required for the two pencils of light to pass through the liquid will be $\dfrac{l}{u - \theta x} - \dfrac{l}{u + \theta x} = \dfrac{2l\theta x}{u^2}$ very nearly. If $\varDelta$ is the double distance traveled in this time in air, in terms of λ, the wave-length, then

$$\varDelta = \frac{4l\theta n^2 x}{\lambda v} \text{ whence } x = \frac{\lambda v}{4ln^2\theta}\, \varDelta.$$

λ was taken as ·00057 cm.

$v =$ 30000000000 cm.

$n^2 =$ 1·78.

The length l was obtained as follows : The stream entered each tube by two tubes a, b (figs. 1, 2) and left by two similar ones d, c. The beginning of the column was taken as the intersection, o, of the axes of a and b, and the end, as the intersection, o', of the axes of d and c. Thus $l = oo'$. $\varDelta$ is found by observing the displacement of the fringes; since a displacement of one whole fringe corresponds to a difference of path of one whole wave-length.

Observations of the double displacement Δ.

1st Series. l = 3·022 meters.
θ = 8·72 meters per second.
Δ = double displacement; w = weight of observation.

Δ.	w.	Δ.	w.	Δ.	w.	Δ.	w.
.510	1·9	·521	0·9	·529	0·6	·515	2·5
·508	1·6	·515	·9	·474	2·0	·525	2·7
·504	1·7	·575	·6	·508	1·4	·480	·8
·473	1·4	·538	2·1	·531	·8	·493	10·6
·557	·4	·577	·6	·500	·5	·348	2·8
·425	·6	·464	1·7	·478	·6	·399	6·7
·560	2·8	·515	1·2	·499	1·0	·482	2·1
·544	·1	·460	·4	·558	·4	·472	2·0
·521	·1	·510	·5	·509	2·0	·490	·8
·575	·1	·504	·5	·470	2·1		

2d Series. l = 6·151, θ = 7·65.

Δ.	w.	Δ.	w.	Δ.	w.	Δ.	w.
·789	4·9	·891	1·7	·909	1·0	·882	6·6
·780	3·5	·883	2·5	·899	1·7	·908	5·9
·840	4·6	·852	11·1	·832	4·3	·965	2·0
·633	1·1	·863	1·5	·837	2·1	·967	3·3
·876	7·3	·843	1·1	·848	1·9		
·956	3·6	·820	3·4	·977	4·7		

3d Series. l = 6·151, θ = 5·67.

Δ.	w.	Δ.	w.	Δ.	w.	Δ.	w.
·640	4·4	·626	11·3	·636	3·1	·619	6·5

If these results be reduced to what they would be if the tube were 10ᵐ long and the velocity 1ᵐ per second, they would be as follows:

Series.	Δ.
1	·1858
2	·1838
3	·1800

The final weighted value of Δ for all observations is Δ = ·1840. From this, by substitution in the formula, we get

$$x = ·434 \text{ with a possible error of } \pm ·02.$$
$$\frac{n^2 - 1}{n^2} = ·437.$$

The experiment was also tried with air moving with a velocity of 25 meters per second. The displacement was about $\frac{1}{100}$ of a fringe; a quantity smaller than the probable error of observation. The value calculated from $\frac{n^2 - 1}{n^2}$ would be ·0036.

It is apparent that these results are the same for a long or short tube, or for great or moderate velocities. The result was also found to be unaffected by changing the azimuth of the fringes to 90°, 180° or 270°. It seems extremely improbable that this could be the case if there were any serious constant error due to distortions, etc.

The result of this work is therefore that the result announced by Fizeau is essentially correct; and that *the luminiferous ether is entirely unaffected by the motion of the matter which it permeates.*