An Improved Test of the Isotropy of Space Using Laser Techniques

A. Brillet[1] and J.L. Hall[2]

Joint Institute for Laboratory Astrophysics,
National Bureau of Standards and University of Colorado
Boulder, CO 80309 USA

This paper describes a new laser version of the Michelson-Morley [1] experiment, in which we achieved a sensitivity $\Delta c/c = (1.5 \pm 2.5) \times 10^{-15}$ [2]. This appears to constitute the most precise test of special relativity yet realized.

Experimental tests of special relativity have been numerous, and often difficult to compare as concerns their meaning and accuracy. Basically, to make such comparisons clear, one needs to have in mind a model of a possible type of failure of the conventional theory. What is needed is a "theory of all possible theories" so that, for example, the Lorentz-Einstein special theory of relativity represents just one point in a suitable hyperspace. Experiments would then serve the role of localizing acceptable theories to the vicinity of this point. However such a more general theory is difficult to invent if it is to include electromagnetic phenomena in a self-consistent way and still be a generalization of the Lorentz transform. Thus up to now most progress has been made by restricting attention to kinematical phenomena. Useful conceptual advances were made in the postwar period by ROBERTSON [3] along these lines. More recently, theoretical papers by MANSOURI and SEXL [4] have appeared proposing a "Test Theory of Special Relativity" which expands Robertson's ideas in a concrete and useful way. This new theory considers (small) additional velocity dependences beyond the Lorentz transformation and is therefore able to make connection and comparisons between previously unrelated experimental "tests of special relativity." Its particular strength is that it enables one to assess the power of the numerous experiments to restrict the acceptable domain in the parameter hyperspace mentioned before. As this theory of Mansouri and Sexl restricts itself to kinematic aspects only and becomes cumbersome for very high velocities, it cannot be regarded as a permanent, complete test theory. But it does show clearly the significance of the several classical relativity experiments and helps one to see what kind of new measurements are needed. Thus it is useful for us to present its basic outline here in a simplified form.

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[1]Permanent Address: Lab. de l'Horloge Atomique, Orsay, France. JILA Visitor, 1978-79 with partial support by a NATO fellowship.

[2]Staff Member, Quantum Physics Division, National Bureau of Standards.

Consider the following assumptions,

1) Existence of a preferred reference frame Σ in which there is no preferred direction.

2) In Σ the speed of electromagnetic waves is c, independent of direction, wavelength, source motion ... .

3) Existence of a second frame S moving with the velocity v along the x axis of Σ.

4) The coordinate transformation between Σ and S is linear.

After a few simplifications relating to orientation of the axes, the general form of the coordinate transformation can be shown to be [4]

$$
\begin{aligned}
t &= a(v)T + \varepsilon x + \varepsilon' y + \varepsilon' z \\
x &= b(v)(X - vT) \\
y &= d(v)Y \\
z &= d(v)Z
\end{aligned} \qquad (1)
$$

where the capital letters refer to Σ. The parameters ε, ε' are determined by clock synchronization procedures, whereas a(v), b(v) and d(v), which contain the real physics of the problem, are to be determined by experiments or by a more fundamental theory.

The velocity of a light ray propagating in S at an angle θ relative to the x axis is given by [4]

$$
\begin{aligned}
C(\theta) = C * \{&\cos\theta\, \varepsilon b(1-v^2) + va\cos\theta + \varepsilon'(1-v^2)b\sin\theta \\
&- a[\cos^2\theta + b^2d^2(1-v^2)\sin^2\theta]^{1/2}\} * \\
&* \{\cos^2\theta(\varepsilon^2 b - \varepsilon^2 bv^2 - a^2/b + 2\varepsilon va) + \sin^2\theta(\varepsilon'^2 b - v^2\varepsilon'^2 b - \\
&- d^2ba^2) + 2\sin\theta\cos\theta * \varepsilon'(\varepsilon b - v^2\varepsilon b + va)\}^{-1} \quad . \qquad (2)
\end{aligned}
$$

In this equation only we use the notation $v/c \rightarrow v$. Let us use the expansions [5] $a(v) = 1+\alpha(v/c)^2+\ldots$, $b(v) = 1+\beta(v/c)^2+\ldots$, $d(v) = 1+\delta(v/c)^2+\ldots$ (Special relativity corresponds to $\alpha=-1/2$, $\beta=1/2$, $\delta=0$.) As an example of the power and utility of this approach, MANSOURI and SEXL [4] show that the time dilation factor, α, can be determined by first order experiments, using frequency standards synchronized by transport. α can also be deduced from high resolution Mössbauer rotor experiments [6] as well as from direct measurement [7]. β and δ can only be studied through second order experiments. In the case of Einstein's synchronization, one gets to second order:

$$
c/c(\theta) = 1 + (\beta + \delta - \tfrac{1}{2})\left(\tfrac{v}{c}\right)^2 \sin^2\theta + (\alpha - \beta + 1)\left(\tfrac{v}{c}\right)^2 \quad . \qquad (3)
$$

The term $[\beta+\delta-(1/2)]$ is measured by "Michelson-Morley" experiments [1,2] and $(\alpha-\beta+1)$ by "Kennedy-Thorndike" [8] experiments. Assuming the velocity of earth relative to "the preferred frame" to be v = 300 km/s [9], the most precise experiments up to now lead to the following limits:

a) $\alpha = -\frac{1}{2} \pm 10^{-7}$ [6] (Mössbauer rotor)

b) $\beta+\delta = \frac{1}{2} \pm 10^{-5}$ [10] (Michelson Morley)

c) $\alpha-\beta = 1.02 \pm 2 \times 10^{-2}$ [8] (Kennedy-Thorndike) .

Although the determination of $(\alpha-\beta)$ was by far the least precise, we felt more able to improve by a large factor the precision of the measurement of $(\beta+\delta)$, mainly because the $\sin^2\theta$ term allows modulation of the effect by simply rotating the experiment in the laboratory. By contrast, the "Kennedy-Thorndike" term $(\alpha-\beta-1)$ will show its principal variations only with a time scale of a year, and the precision of the measurements is then strongly degraded by low frequency noise (drifts).

Our "spatial isotropy" experiment to determine $[\beta+\delta-(1/2)]$ has been designed to be clear in its interpretation and free of spurious effects. Its principle may be understood by reference to Fig. 1. A He-Ne laser (λ = 3.39 μm) wavelength is servostabilized so that its radiation satisfies optical standing-wave boundary conditions in a highly stable, isolated Fabry-Perot interferometer. Because of the servo, length variations of this cavity -- whether accidental or cosmic -- appear as variations of the laser wavelength. They can be read out with extreme sensitivity as a frequency shift by optically heterodyning a portion of the laser power with another highly stable laser, provided in our case by a CH_4-stabilized [11] laser. To separate a potential cosmic cavity-length variation from simple drift, we arranged to rotate the direction of the cavity length by mounting the length etalon, its laser and optical accessories, onto a 95-cm × 40-cm × 12-cm granite slab which, along with servo and power-supply electronics, may be continuously rotated. (The frequency readout beam comes from a beam splitter up along the rotation axis and is directed over to the CH_4-stabilized laser. Electrical power comes to the rotating table through Hg-filled channels and a contactor pin assembly below the table.) The table rotation angle is sensed via 25 holes pierced in a metal band under the table. A single, separate hole provides absolute resynchronization each turn. The laser beat frequency is counted for 0.2 sec under minicomputer control after each synchronizing pulse, scaled and transfered to storage and display. A genuine spatial anisotropy would be manifest as a beat-frequency variation $\propto P_2(\cos\theta)$. The associated laser-frequency shift may be conveniently expressed as a vector amplitude at twice the table rotation frequency, f, of 1 per ~10 sec. Furthermore, its component in the plane perpendicular to Earth's spin axis should precess 360° in 12 h.

Our fundamental etalon of length is an interferometer which employs fused-silica mirrors "optically contacted" onto a low-expansion glass-ceramic [12] tube of 6-cm o.d. × 1-cm wall × 30.5-cm length. The choice of 50-cm mirror radii provides a well-isolated TEM_{00} mode. Dielectric coatings at the mirrors' centers provide an interferometric efficiency of 25% and a fringe width ≈4.5 MHz. The interferometer mounts inside a massive, thermally isolated Aℓ vacuum envelope. The environmental temperature is stable to 0.2°C, but no further thermal servo was employed.

Fringe distortion due to optical feedback is prevented by a cascade of three yttrium-iron-garnet Faraday isolators, each having a return loss >26 dB. The laser is frequency modulated ≈2.5 MHz peak to peak at 45 kHz. Both first-harmonic and third-harmonic locking were tried, the unused one being a useful diagnostic for adjustment of the Faraday isolators. Based on the 200-μW available fringe signal, the frequency noise of the cavity-stabilized laser is expected (and observed) to be about 20 Hz for a 1-sec measurement, using a first-harmonic lock.

Our CH_4-stabilized "telescope-laser" frequency reference system achieves a comparable stability [11]. The random noise of the beat signal in a typical 20-min data block is observed to be ~3 Hz, compared with the laser frequency of almost 10^{14} Hz. To ensure absolute isolation of the cavity-stabilized and CH_4-stabilized lasers, the latter actually is used to phase lock a "local-oscillator" laser offset by 120 MHz. The ~35-MHz beat of this isolation laser with the cavity-stabilized laser is the measured quantity. See Fig. 1. After each measurement the computer checks that the beat frequency is near its optimum value and readjusts the frequency synthesizer if necessary.

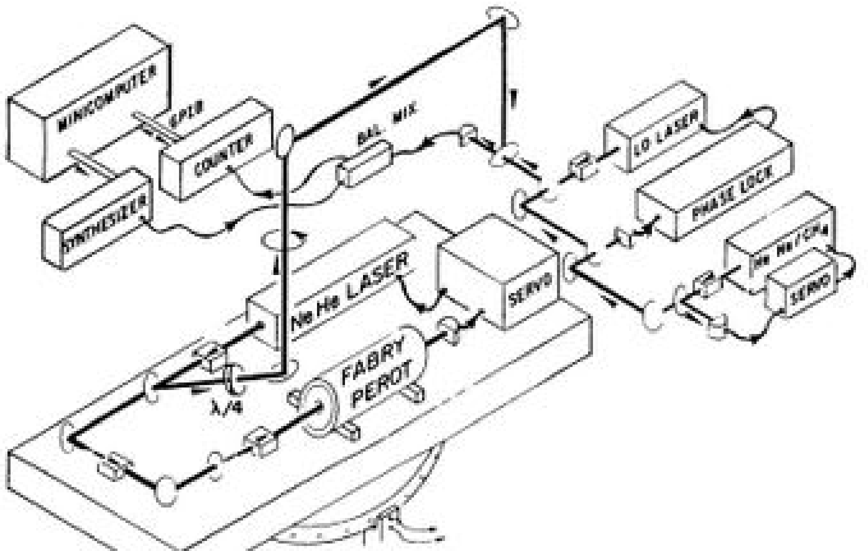

Fig. 1. Schematic of isotropy-of-space experiment. A He-Ne laser (3.39 μm) is servostabilized to a transmission fringe of an isolated and highly stable Fabry-Perot resonator, with provision being made to rotate this whole system. A small portion of the laser beam is diverted up along the table rotation axis to read out the cavity length via optical heterodyne with an "isolation laser" which is stabilized relative to a CH_4-stabilized reference laser. The beat frequency is shifted and counted under minicomputer control, these frequency measurements being synchronized and stored relative to the table's angular position. After 30 minutes of signal averaging the data are Fourier transformed and printed out, and the experiment is reinitialized.

Representative time series data are shown in Fig. 2. In the upper curve one can see a long-term downward trend, which corresponds to a drift rate of ~-50 Hz/sec. The superimposed sinewave has an amplitude of about 200 Hz and a period of 1 cycle per table revolution. It arises from gravitational stretching of the interferometer because the mounting of the interferometer is inevitably somewhat unsymmetrical about its center and the rotation axis is not absolute vertical. As this spurious signal occurs at 1/2 the frequency of the "anisotropy signal," we can reasonably project it out -- along with the drift -- by fitting these terms and subtracting them from the data. The resulting data are plotted in the lower part of Fig. 2 with 20-fold increased sensitivity. Note that another spurious signal is also clearly present, this one at the interesting frequency of 2 cycles per revolution. It is possible that this term arises via nonlinear elastic response to the 1 cycle per revolution perturbation. Centrifugal stretching of the interferometer due to rotation gives -10 kHz at f = (1 turn)/13 sec and implies a compliance ~10 times that of the bulk material. (Thus small synchronous speed variations could also give rise to synchronous acceleration forces.) As will be seen below, the spurious signal at 2 cycles per revolution basically controls the useful sensitivity of our experiment.

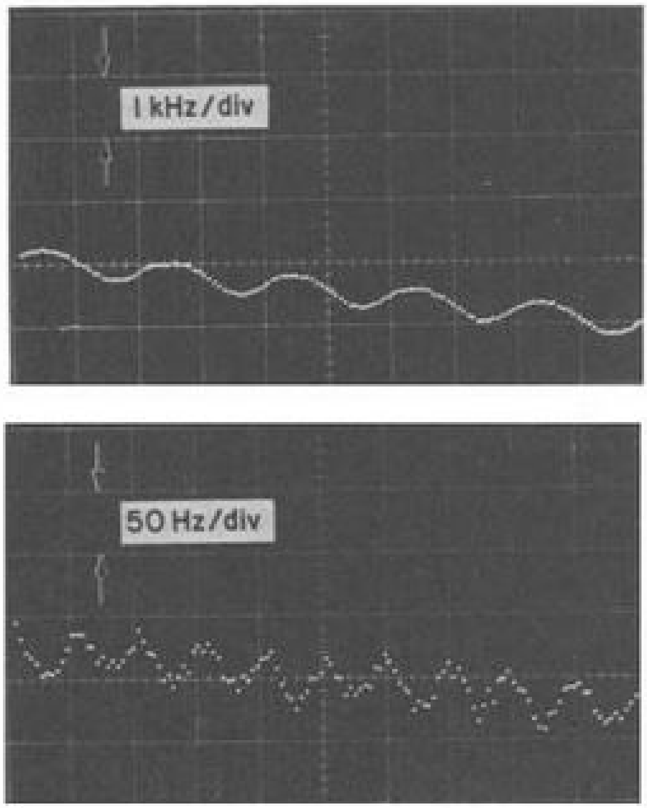

Fig. 2. Upper curve shows time series data for a typical 600 sec run (5 turns per sweep, averaged for 10 sweeps). The general downward trend of the beat frequency represents a drift rate of ~-50 Hz/sec. Superimposed on this trend is a sinusoidal variation of some 200 Hz amplitude with a period of 1 cycle per table revolution. This "sine-wave" signal is caused by a varying gravitational stretching of the interferometer. See text. Lower curve shows data at 20-fold higher sensitivity after this spurious signal and drift have been removed. Note that a signal at the interesting frequency of 2 cycles per revolution is now visible.

We find that taking data in blocks of N table rotations ($N \approx 8$-50) is helpful in minimizing the cross coupling of the first two noise sources into the interesting Fourier bin at 2 cycles per table revolution (actually at 2N cycles per N table revolutions). Typically 10-20 blocks of N revolutions were averaged together in the minicomputer before calculating the amplitude and phase of the signal at the second harmonic of the table rotation frequency. The average result is an amplitude of $\cos 2\theta$ of ≈ 17 Hz (2×10^{-13}) with an approximately constant phase in the laboratory frame. A number of such 1/2-h averages spanning a 24-h period are illustrated in Fig. 3 as radius vectors from the origin to the open circles. The noise level of each such average was estimated by computing the noise at the nearby Fourier bins of $2N \pm 1$ cycles per N table revolutions. For a 1/2-h average (N = 10, averaged 10 times) the typical noise amplitude was 2 Hz with a random phase.

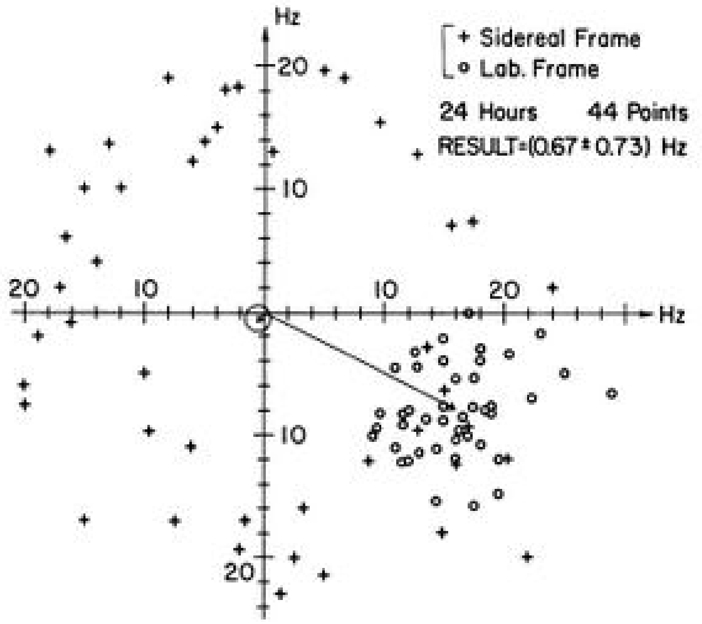

Fig. 3. Second Fourier amplitude from one day's data. The vector Fourier component at twice the table rotation rate is plotted as the radius vector from the origin to the open circles. After precessing these vectors by their appropriate sidereal angles they are plotted as the (+). For the 24-h block of data the average "ether drift" term is 0.67 ± 0.73 Hz, corresponding to $\Delta\nu/\nu = (0.76 \pm 0.83) \times 10^{-14}$.

To discriminate between this persistent spurious signal (17-Hz amplitude at 2f) and any genuine "ether" effect, we made measurements for 12 or 24 sidereal hours. We must rotate each vector to obtain its phase relative to a fixed sidereal axis prior to further averaging. Averaging after this rotation leads, as shown in Fig. 3, to a typical 1-day result below 1 ± 1 Hz. We believe Fig. 3 makes a convincing case that the origin of our spurious $\cos 2\theta$ "signal" is to be sought in the domain of laboratory physics. Further experiments are underway to test the nonlinear elasticity conjecture, as well as to measure the paramagnetic and gravitational gradient contributions. A number of 12- and/or 24-h averages are shown in Fig. 4. We felt that averages for 24 h were sometimes quieter than 12-h

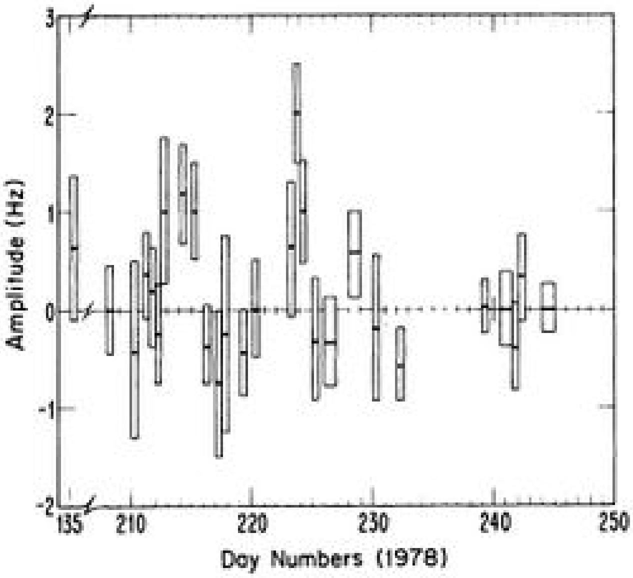

Fig. 4. Averaged data of isotropy-of-space experiment. Data such as those in Fig. 2 were averaged in blocks of 12 h (thinner bars) or 24 h (thicker bars). For completeness this figure includes data from diagnostic experiments before day 225. The data after day 238 represent near-ideal automatic operation of the present apparatus. A 1-Hz amplitude represents $\sim 1.1 \times 10^{-14}$ fractional frequency shift. The reference axis for the projection is the direction identified by Smoot et al.[11.0-h R.A. (right ascension, 6° dec. (declination)], Ref. 9.

averages, an effect which may be related to the observed 24-h period of the floor tilt ($\approx\mu$rad). The data in Fig. 4 include most of the points taken during various diagnostic experiments. The data taken after day 238 correspond to approximately "ideal" automated operation of the present apparatus. The lack of any significant signal or day dependence allows us to perform an overall average. This final result of our experiment is a null "ether drift" of 0.13±0.22 Hz, which represents a fractional frequency shift of $(1.5 \pm 2.5) \times 10^{-15}$.

From Eq. (3), we get $[\beta+\delta-(1/2)](v/c)^2 = (1.5 \pm 2.5) \times 10^{-15}$. This limit represents a 4000-fold improvement over the most sensitive previous experiment [10]. This advance is due to smaller spurious signals in our experiment (2×10^{-13} instead of 10^{-9}), to superior data-processing techniques and to superior long-term stability of the length etalon and reference laser.

To evaluate the precision with which this result verifies special relativity, one has to make assumptions concerning the direction and the amplitude of $\vec{v}$, the velocity of earth relative to "the preferred frame." We may conservatively use the Earth's velocity around the sun, which gives a null result $(\beta + \delta - 1/2) = \pm 5 \times 10^{-7}$, some 10^6 times smaller than the classical prediction. This result includes the the sensitivity reduction factor 0.43 × associated with our 40° latitude and the data processing technique we used. (This factor had always been overlooked by previous authors.)

The recent discovery of a pure $P_1(\cos\theta)$ anisotropy in the cosmic blackbody radiation [9] was interpreted as a Doppler shift produced by motion of the earth (400 km/s) relative to the "preferred inertial frame" in which the blackbody radiation is isotropic. If this velocity is considered to be the relevant one, our sensitivity is $\beta + \delta - 1/2 = \pm 2.5 \times 10^{-9}$ and constitutes the most precise test yet of the Lorentz transformation.

The present sensitivity limit arises from two sources: the finite averaging time and some mechanical problems. To improve our result another decade by simple averaging would require 15 years. The same improvement should be possible in a few months averaging with improved mechanical design (rotation speed stabilized to 10^{-4}, better design and mounting of the cavity, taking into account nonlinear elastic mechanical response, and active computer-driven piezo stabilization of the rotation axis to 1 arc sec). plus better vacuum inside the interferometer, and more effective stabilization of its housing temperature. Given success in reducing the systematic effects, higher laser power and better detectors could be used to reduce the random noise level which is presently far above the shot noise limit.

Further extensions of this experiment may diverge in two different ways:

- An increase in sensitivity by two orders of magnitude from the present level, which would require all the above improvements plus somewhat better laser stability, would certainly be a useful advance in experimental tests of special relativity. It also would bring us to the domain where general relativistic effects may be observable, at least in principle. For example the gradient of the earth's gravity over the size of our apparatus produces a vertical spatial anisotropy of the type we could measure. The expected frequency shift would be $\Delta\nu/\nu \approx \frac{1}{2}(GM/R)(1/c^2)(L/R) \approx 3 \times 10^{-17}$, if the reference etalon were rotated from a horizontal to a vertical position [13]. However it is evident that the gravity force itself would produce a large and probably disastrous background signal. Thus we presume that improved laboratory experiments of the present type will be basically interesting as useful advances in testing the assumptions of special relativity.

However the high resolution length readout techniques themselves may be of interest for gravity wave detectors and other highly sensitive applications as being perhaps more suitable than simple optical interferometry to read out with extreme resolution and dynamic range the positions of two free test masses.

In the other direction, it is attractive to consider applying these high sensitivity readout techniques to a modern Kennedy-Thorndike type of experiment. Improvement by a large factor, say 1000-fold, would certainly be interesting to relativity theorists. However, careful study of Ref. 8 suggests that this level of improvement will be very difficult to attain, although more modest improvements may surely be expected. Extreme care would be necessary to avoid drifts since, as we noted earlier, the only modulation arises from the earth's motion. For example, temperature stability of 50 μK for 1 month and a variation of the creep rate below 10^{-11} per day^2 would allow a ~20-fold improvement on the results of Ref. 8.

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13. One can understand the expected anisotropy as a manifestation of the gravitational redshift with height or, equivalently, in terms of a time delay of light signals propagating through a gravitational gradient. Direct measurements of the latter type, based on tracking data from the Mars-Viking lander, have recently measured this term to 0.2% [14].
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