Ralph Malcolm Rabbidge's thought experiment

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1 Introduction

Ralph Malcolm Rabbidge, who is posting in the usenet-group sci.physics.relativity under the pseudonym Henry Wilson, has presented a thought experiment for which he claims that the Special Theory of Relativity (hereafter called SR) will give contradictory predictions depending on in which frame of reference the calculations of the predictions are made.

I will show that this is not the case.

Ralph Malcolm Rabbidge's original post

2 The thought experiment

2.1 Definition of the thought experiment

Three clocks, A, B and C are initially stationary on a rod with length L. C is in the middle of the rod, A and B at the ends. The three clocks are synchronized in the rest frame of the rod according to Einstein's method.

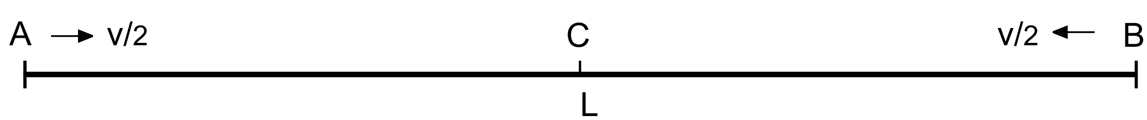

Figure 1: *The rod with the clocks*

When the clocks all show 0, A and B are instantly set in motion towards each other, each with the speed $\frac{v}{2}$ relative to the rod. C remains stationary at the middle of the rod.

Let E_A be the event that A is at the end of the rod, showing 0, and starts moving.
Let E_B be the event that B is at the other end of the rod, showing 0, and starts moving.
Let E_C be the event that the two clocks meet at the middle of the rod.

The problem is to find what the three clocks show when they meet.

Let t_1 be what clock A shows when it reaches clock C at event E_C .
Let t_2 be what clock B shows when it reaches clock C at event E_C .
Let t_0 be what clock C shows at event E_C .

2.2 Calculation of t_0, t_1 and t_2

Let K be the rest frame of the rod, and let K' be the rest frame of clock A.
Let us choose coordinate systems as shown in fig.2.

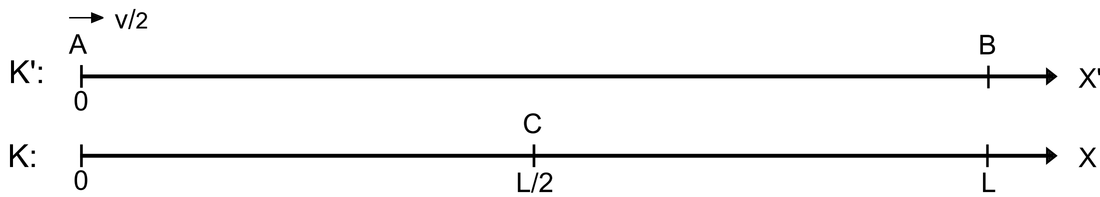

Figure 2: *The frames of reference K and K'*

2.2.1 Calculation of t_0 and t_1 in the rest frame of the rod

Clock A, which is moving the distance $\frac{L}{2}$ at the speed $\frac{v}{2}$, will reach clock C when the latter shows:

$$t_0 = \frac{\frac{L}{2}}{\frac{v}{2}} = \frac{L}{v}$$

Clock A is showing zero at event E_A, and is running slow by the factor $\sqrt{1-\frac{(\frac{v}{2})^2}{c^2}}$ as observed in frame K. At event E_C clock A will thus show:

$$t_1 = \frac{\frac{L}{2}}{\frac{v}{2}}\sqrt{1-\frac{(\frac{v}{2})^2}{c^2}} = \frac{L}{v}\sqrt{1-\frac{v^2}{4c^2}}$$

A more proper way to find t_1 is however to transform the coordinates of event E_C to K', the rest frame of clock A.

The coordinates of the event E_C in frame K are: $x_C = \frac{L}{2}$, $t_C = t_0 = \frac{L}{v}$

The coordinates of the event E_C transformed to K' are:

$$x'_C = \frac{x_C - \frac{v}{2}t_C}{\sqrt{1-\frac{(\frac{v}{2})^2}{c^2}}} = \frac{\frac{L}{2} - \frac{v}{2}\frac{L}{v}}{\sqrt{1-\frac{v^2}{4c^2}}} = 0$$

$$t'_C = \frac{t_C - \frac{\frac{v}{2}x_C}{c^2}}{\sqrt{1-\frac{(\frac{v}{2})^2}{c^2}}} = \frac{\frac{L}{v} - \frac{\frac{v}{2}\frac{L}{2}}{c^2}}{\sqrt{1-\frac{v^2}{4c^2}}} = \frac{L}{v}\sqrt{1-\frac{v^2}{4c^2}}$$

t'_C is the proper time of clock A, so $t_1 = t'_C = \frac{L}{v}\sqrt{1-\frac{v^2}{4c^2}}$

2.2.2 Calculation of t_1 and t_2 in the rest frame of clock A

The coordinates of the event E_B in frame K are: $x_B = L$, $t_B = 0$

The coordinates of the event E_B transformed to K' are:

$$x'_B = \frac{x_B - \frac{v}{2}t_B}{\sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}}} = \frac{L}{\sqrt{1 - \frac{v^2}{4c^2}}}$$

$$t'_B = \frac{t_B - \frac{\frac{v}{2}x_B}{c^2}}{\sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}}} = -\frac{L}{2v}\frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{4c^2}}}$$

Clock B will be moving towards A with the speed w:

$$w = \frac{\frac{v}{2} + \frac{v}{2}}{1 + \frac{\frac{v}{2}\frac{v}{2}}{c^2}} = \frac{v}{1 + \frac{v^2}{4c^2}}$$

Clock B starts at the time t'_B and is moving the distance x'_B with the speed w, so when clock B meets clock A, the latter will show:

$$t_1 = t'_B + \frac{x'_B}{w} = -\frac{L}{2v}\frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{4c^2}}} + \frac{\frac{L}{\sqrt{1 - \frac{v^2}{4c^2}}}}{\frac{v}{1 + \frac{v^2}{4c^2}}} = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

Clock B will start moving (showing 0) from the position x'_B, so it will move the distance x'_B with the speed w.

As observed in frame K', clock B will run slow by the factor $\sqrt{1 - \frac{w^2}{c^2}} = \frac{1 - \frac{v^2}{4c^2}}{1 + \frac{v^2}{4c^2}}$.

When clock B meets clock A, the former will thus show:

$$t_2 = \frac{x'_B}{w}\left(\frac{1 - \frac{v^2}{4c^2}}{1 + \frac{v^2}{4c^2}}\right) = \left(\frac{\frac{L}{\sqrt{1 - \frac{v^2}{4c^2}}}}{\frac{v}{1 + \frac{v^2}{4c^2}}}\right)\left(\frac{1 - \frac{v^2}{4c^2}}{1 + \frac{v^2}{4c^2}}\right) = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

2.3 Alternative calculation of t_0, t_1 and t_2

Let M be the rest frame of the rod, and let M' be the rest frame of clock B.
Let us choose coordinate systems as shown in fig.3.

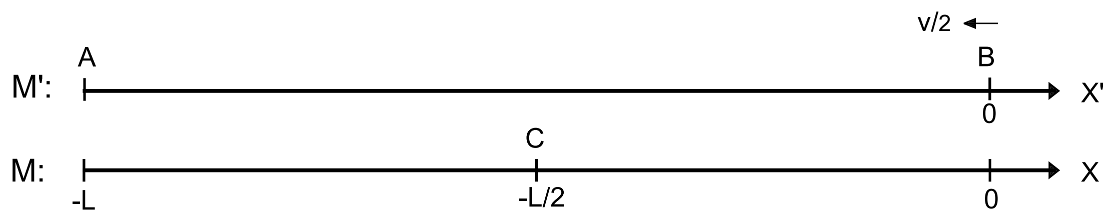

Figure 3: *The frames of reference M and M'*

2.3.1 Calculation of t_0 and t_2 in the rest frame of the rod

Clock B, which is moving the distance $\frac{L}{2}$ at the speed $\frac{v}{2}$, will reach clock C when the latter shows:

$$t_0 = \frac{\frac{L}{2}}{\frac{v}{2}} = \frac{L}{v}$$

Clock B is showing zero at event E_B, and is running slow by the factor $\sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}}$ as observed in frame M. At event E_C clock B will thus show:

$$t_2 = \frac{\frac{L}{2}}{\frac{v}{2}} \sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}} = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

A more proper way to find t_2 is however to transform the coordinates of event E_C to M', the rest frame of clock B.

The coordinates of the event E_C in frame M are: $x_C = -\frac{L}{2}$, $t_C = t_0 = \frac{L}{v}$

The coordinates of the event E_C transformed to M' are:

$$x'_C = \frac{x_C + \frac{v}{2}t_C}{\sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}}} = \frac{-\frac{L}{2} + \frac{v}{2}\frac{L}{v}}{\sqrt{1 - \frac{v^2}{4c^2}}} = 0$$

$$t'_C = \frac{t_C + \frac{\frac{v}{2}x_C}{c^2}}{\sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}}} = \frac{\frac{L}{v} - \frac{\frac{v}{2}\frac{L}{2}}{c^2}}{\sqrt{1 - \frac{v^2}{4c^2}}} = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

t'_C is the proper time of clock B, so $t_2 = t'_C = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$

2.3.2 Calculation of t_1 and t_2 in the rest frame of clock B

The coordinates of the event E_A in frame M are: $x_A = -L$, $t_A = 0$

The coordinates of the event E_A transformed to M' are:

$$x'_A = \frac{x_A + \frac{v}{2} t_A}{\sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}}} = \frac{-L}{\sqrt{1 - \frac{v^2}{4c^2}}}$$

$$t'_A = \frac{t_A + \frac{\frac{v}{2} x_A}{c^2}}{\sqrt{1 - \frac{(\frac{v}{2})^2}{c^2}}} = -\frac{L}{2v} \frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{4c^2}}}$$

Clock A will be moving towards B with the speed w:

$$w = \frac{\frac{v}{2} + \frac{v}{2}}{1 + \frac{\frac{v}{2}\frac{v}{2}}{c^2}} = \frac{v}{1 + \frac{v^2}{4c^2}}$$

Clock A starts at the time t'_A and is moving the distance $(0 - x'_A)$ with the speed w, so when clock A meets clock B, the latter will show:

$$t_2 = t'_A + \frac{0 - x'_A}{w} = -\frac{L}{2v} \frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{4c^2}}} + \frac{\frac{L}{\sqrt{1 - \frac{v^2}{4c^2}}}}{\frac{v}{1 + \frac{v^2}{4c^2}}} = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

Clock A will start moving (showing 0) from the position x'_A, so it will move the distance $(0 - x'_A)$ with the speed w.
As observed in frame M', clock A will run slow by the factor $\sqrt{1 - \frac{w^2}{c^2}} = \frac{1 - \frac{v^2}{4c^2}}{1 + \frac{v^2}{4c^2}}$.

When clock A meets clock B, the former will thus show:

$$t_1 = \frac{0 - x'_A}{w}\left(\frac{1 - \frac{v^2}{4c^2}}{1 + \frac{v^2}{4c^2}}\right) = \left(\frac{\frac{L}{\sqrt{1 - \frac{v^2}{4c^2}}}}{\frac{v}{1 + \frac{v^2}{4c^2}}}\right)\left(\frac{1 - \frac{v^2}{4c^2}}{1 + \frac{v^2}{4c^2}}\right) = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

3 Conclusion

We have shown that when calculated in the rest frame of the rod, SR predicts that $t_0 = \frac{L}{v}$ and $t_1 = t_2 = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$.

We have shown that when calculated in the rest frame of clock A, SR predicts that $t_1 = t_2 = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$.

We have shown that when calculated in the rest frame of clock B, SR predicts that $t_1 = t_2 = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$.

Ralph Malcolm Rabbidge's claim that the Special Theory of Relativity will give contradictory predictions depending on in which frame of reference the calculations of the predictions are made, is therefore false.

4 Addendum April 27, 2011

4.1 Modified scenario

We will assume that clock A and B, at the events E_A and E_B respectively, begin transmitting a radio signal with frequency f.

In the rest frame of clock A we will calculate how many cycles clock A transmits before clock A and B are co-located, and we will calculate how many cycles clock A will receive from clock B.

4.2 Previous results

From section 2.2.2 we have:

The coordinates of the event E_B in the rest frame of clock A are:

$$x'_B = \frac{L}{\sqrt{1 - \frac{v^2}{4c^2}}}$$

$$t'_B = -\frac{L}{2v} \frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{4c^2}}}$$

Clock B will be moving towards A with the speed w:

$$w = \frac{v}{1 + \frac{v^2}{4c^2}}$$

When clock B meets clock A, the latter will show:

$$t_1 = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

When clock B meets clock A, the former will show:

$$t_2 = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

As observed in the rest frame of clock A, clock B will run slow by the factor $\sqrt{1 - \frac{w^2}{c^2}} = \frac{1 - \frac{v^2}{4c^2}}{1 + \frac{v^2}{4c^2}}$.

This could also be calculated thus:

$$\frac{t_2 - 0}{t_1 - t'_B} = \frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

4.3 Counting cycles

The number of cycles N_A transmitted by clock A before the clocks are co-located is:

$$N_A = f\,(t_1 - 0) = f\frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

Clock A will receive the first cycle transmitted from clock B at the time t'_B plus the time the signal uses to reach A. Let's call this time t_f.

$$t_f = t'_B + \frac{x'_B}{c} = -\frac{L}{2v}\frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{4c^2}}} + \frac{L}{c}\frac{1}{\sqrt{1 - \frac{v^2}{4c^2}}} = \frac{L}{c}\left(\frac{1 - \frac{v}{2c}}{\sqrt{1 - \frac{v^2}{4c^2}}}\right)$$

The number of cycles N_B received by clock A before the clocks are co-located is:

$$N_B = f'\,(t_2 - t_f) = f'\left(\frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}} - \frac{L}{c}\left(\frac{1 - \frac{v}{2c}}{\sqrt{1 - \frac{v^2}{4c^2}}}\right)\right) = f'\frac{L}{v}\left(\frac{1 + \frac{v^2}{4c^2} - \frac{v}{c}}{\sqrt{1 - \frac{v^2}{4c^2}}}\right)$$

where f' is the Doppler shifted frequency received by A.

$$f' = f\sqrt{\frac{1 + \frac{w}{c}}{1 - \frac{w}{c}}} = f\sqrt{\frac{1 + \frac{v^2}{4c^2} + \frac{v}{c}}{1 + \frac{v^2}{4c^2} - \frac{v}{c}}}$$

The number of cycles N_B received by clock A is thus:

$$N_B = f\left(\sqrt{\frac{1 + \frac{v^2}{4c^2} + \frac{v}{c}}{1 + \frac{v^2}{4c^2} - \frac{v}{c}}}\right)\left(\frac{L}{v}\right)\left(\frac{1 + \frac{v^2}{4c^2} - \frac{v}{c}}{\sqrt{1 - \frac{v^2}{4c^2}}}\right) = f\frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$$

4.4 Conclusion

Clock B will run slow by the factor $\frac{1-\frac{v^2}{4c^2}}{1+\frac{v^2}{4c^2}}$, as observed in the rest frame of clock A.

The number of cycles N_A transmitted by clock A before the clocks are co-located is equal to the number of cycles N_B emitted by B and received by clock A. $N_A = N_B = f\frac{L}{v}\sqrt{1 - \frac{v^2}{4c^2}}$