# How light is reflected off moving mirrors

## *Basic aberration*

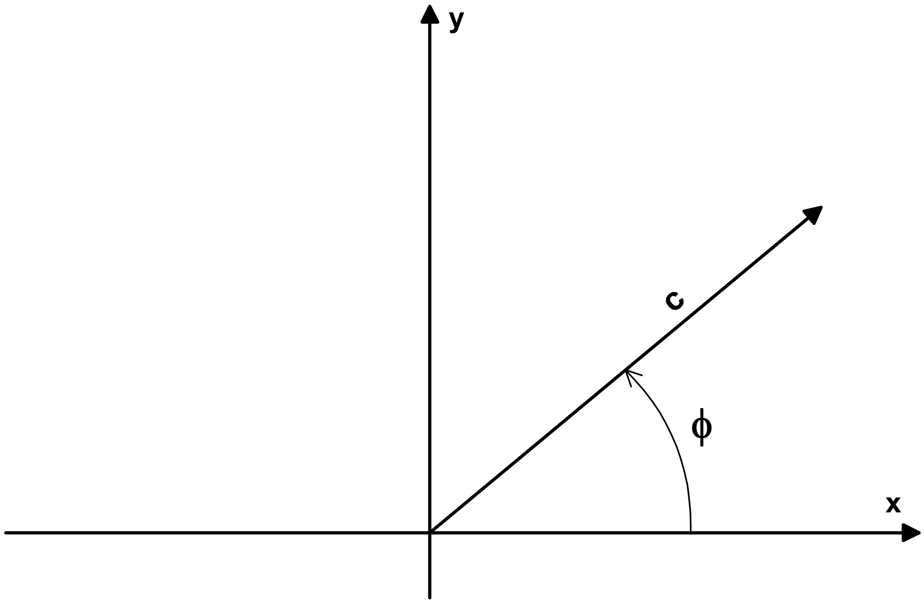


*Fig 1*

In an inertial frame K, a light beam is moving at the speed c, and has an angle ϕ to the x-axis, see fig.1. Another inertial frame K', with the x' and y' axes aligned with the x and y axes of K, is moving with the speed v along the x-axis of K.

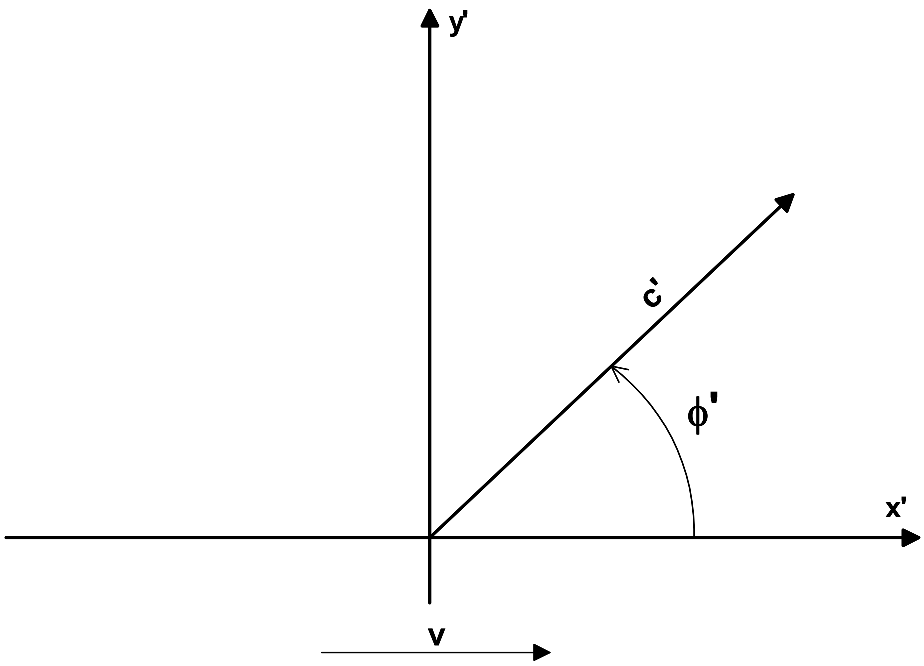


*Fig.2*

### *Lorentz transformation* of the velocity vector yields:

$$\cos(\phi') = \frac{\cos(\phi) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos(\phi)} \qquad\qquad c' = c$$

The inverse transform:

$$\cos(\phi) = \frac{\cos(\phi') + \frac{v}{c}}{1 + \frac{v}{c}\cdot\cos(\phi')} \qquad\qquad c = c'$$

### *Galilean transformation* of the velocity vector yields:

$$\cos(\phi') = \frac{\cos(\phi) - \frac{v}{c}}{\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos(\phi) + \left(\frac{v}{c}\right)^2}} \qquad\qquad c' = c\cdot\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos(\phi) + \left(\frac{v}{c}\right)^2}$$

The inverse transform:

$$\cos(\phi) = \frac{\cos(\phi') + \frac{v}{c'}}{\sqrt{1 + 2\cdot\frac{v}{c'}\cdot\cos(\phi') + \left(\frac{v}{c'}\right)^2}} \qquad\qquad c = c'\cdot\sqrt{1 + 2\cdot\frac{v}{c'}\cdot\cos(\phi') + \left(\frac{v}{c'}\right)^2}$$

A first order approximation in v/c of the Galilean transform is:

$$\cos(\phi') \approx \frac{\cos(\phi) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos(\phi)} \qquad\qquad c' = c - v\cdot\cos(\phi)$$

The inverse transform:

$$\cos(\phi) \approx \frac{\cos(\phi') + \frac{v}{c'}}{1 + \frac{v}{c'}\cdot\cos(\phi')} \qquad\qquad c = c' + v\cdot\cos(\phi')$$

## *Aberration of a light beam reflected off a moving mirror*

A mirror in the x-z plane is stationary in the inertial frame K.

We will take for granted that the "Law of Reflection" applies in K, that is, a light beam incident upon a mirror will be reflected at an angle equal to the incident angle.

According to the Special Theory of Relativity, hereafter called SR, the speed of both the incident light and the reflected light is c, where c is an invariant constant of nature.

According to Ritz Emission Theory, hereafter called RT, the speed of the reflected light is the same as the speed of the incident light in K.

**Case #1, the direction of the light beam is opposite to the motion of mirror:**

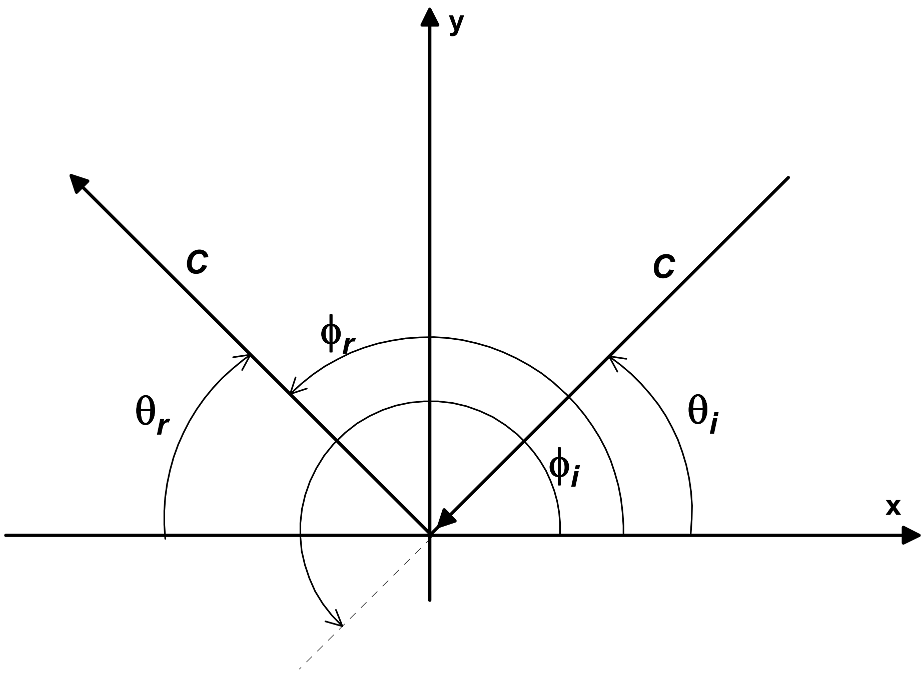


*Fig. 3*

According to the law of reflection $\theta_r = \theta_i$.

A second inertial frame K', with the x' and y' axes aligned with the x and y axes of K, is moving with the speed v along the x-axis of K.

We will find how the velocities of the two beams transform to the K' frame.
According to SR, velocities transform according to the Lorentz transform.
According to RT, velocities transform according to the Galilean transform.

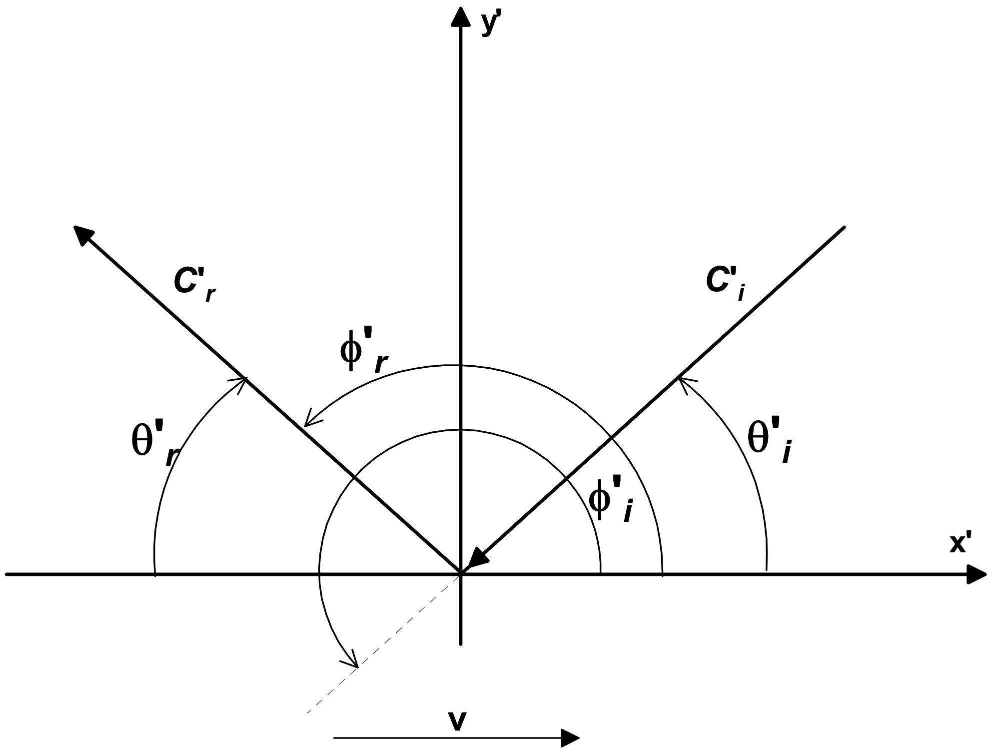


*Fig. 4*

**According to the Lorentz transform, we have:**

$$\cos\left(\phi'_i\right) = \frac{\cos\left(\phi_i\right) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos\left(\phi_i\right)} \qquad \cos\left(\phi'_r\right) = \frac{\cos\left(\phi_r\right) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos\left(\phi_r\right)} \qquad c'_i = c'_r = c$$

$\phi_i = \pi + \theta_i$ and $\phi'_i = \pi + \theta'_i$, hence $\cos(\phi_i) = -\cos(\theta_i)$ and $\cos(\phi'_i) = -\cos(\theta'_i)$

$\phi_r = \pi - \theta_r$ and $\phi'_r = \pi + \theta'_r$, hence $\cos(\phi_r) = -\cos(\theta_r)$ and $\cos(\phi'_r) = -\cos(\theta'_r)$

Hence: $\cos\left(\theta'_i\right) = \dfrac{\cos\left(\theta_i\right) + \frac{v}{c}}{1 + \frac{v}{c}\cdot\cos\left(\theta_i\right)}$ and $\cos\left(\theta'_r\right) = \dfrac{\cos\left(\theta_r\right) + \frac{v}{c}}{1 + \frac{v}{c}\cdot\cos\left(\theta_r\right)}$

Since $\theta_r = \theta_i$, we get: $\cos\left(\theta'_r\right) = \dfrac{\cos\left(\theta_i\right) + \frac{v}{c}}{1 + \frac{v}{c}\cdot\cos\left(\theta_i\right)} = \cos\left(\theta'_i\right)$

Thus $\theta'_r = \theta'_i$

**A light beam incident upon a moving mirror will be reflected at an angle equal to the incident angle.**

**According to the Galilean transform, we have:**

$$\cos(\phi'_i) = \frac{\cos(\phi_i) - \frac{v}{c}}{\sqrt{1 - 2 \cdot \frac{v}{c} \cdot \cos(\phi_i) + \left(\frac{v}{c}\right)^2}} \qquad \cos(\phi_r') = \frac{\cos(\phi_r) - \frac{v}{c}}{\sqrt{1 - 2 \cdot \frac{v}{c} \cdot \cos(\phi_r) + \left(\frac{v}{c}\right)^2}}$$

$$c'_i = c \cdot \sqrt{1 - 2 \cdot \frac{v}{c} \cdot \cos(\phi_i) + \left(\frac{v}{c}\right)^2} \qquad c'_r = c \cdot \sqrt{1 - 2 \cdot \frac{v}{c} \cdot \cos(\phi_r) + \left(\frac{v}{c}\right)^2}$$

Since $\cos(\phi_i) = -\cos(\theta_i)$, $\cos(\phi'_i) = -\cos(\theta'_i)$, $\cos(\phi_r) = -\cos(\theta_r)$ and $\cos(\phi'_r) = -\cos(\theta'_r)$, we find:

$$\cos(\theta'_i) = \frac{\cos(\theta_i) + \frac{v}{c}}{\sqrt{1 + 2 \cdot \frac{v}{c} \cdot \cos(\theta_i) + \left(\frac{v}{c}\right)^2}} \qquad \cos(\theta_r') = \frac{\cos(\theta_r) + \frac{v}{c}}{\sqrt{1 + 2 \cdot \frac{v}{c} \cdot \cos(\theta_r) + \left(\frac{v}{c}\right)^2}}$$

$$c'_i = c \cdot \sqrt{1 + 2 \cdot \frac{v}{c} \cdot \cos(\theta_i) + \left(\frac{v}{c}\right)^2} \qquad c'_r = c \cdot \sqrt{1 + 2 \cdot \frac{v}{c} \cdot \cos(\theta_r) + \left(\frac{v}{c}\right)^2}$$

Since $\theta_r = \theta_i$, we get: $\cos(\theta'_r) = \dfrac{\cos(\theta_i) + \frac{v}{c}}{\sqrt{1 + 2 \cdot \frac{v}{c} \cdot \cos(\theta_i) + \left(\frac{v}{c}\right)^2}} = \cos(\theta'_i)$

Thus $\theta'_r = \theta'_i$ and $c'_r = c'_i$.

**A light beam incident upon a moving mirror will be reflected at an angle equal to the incident angle.**

**The speed of a light beam reflected off a moving mirror will be the same as the speed of the incident beam.**

**Case #2, the direction of the light beam is the same as the motion of mirror:**

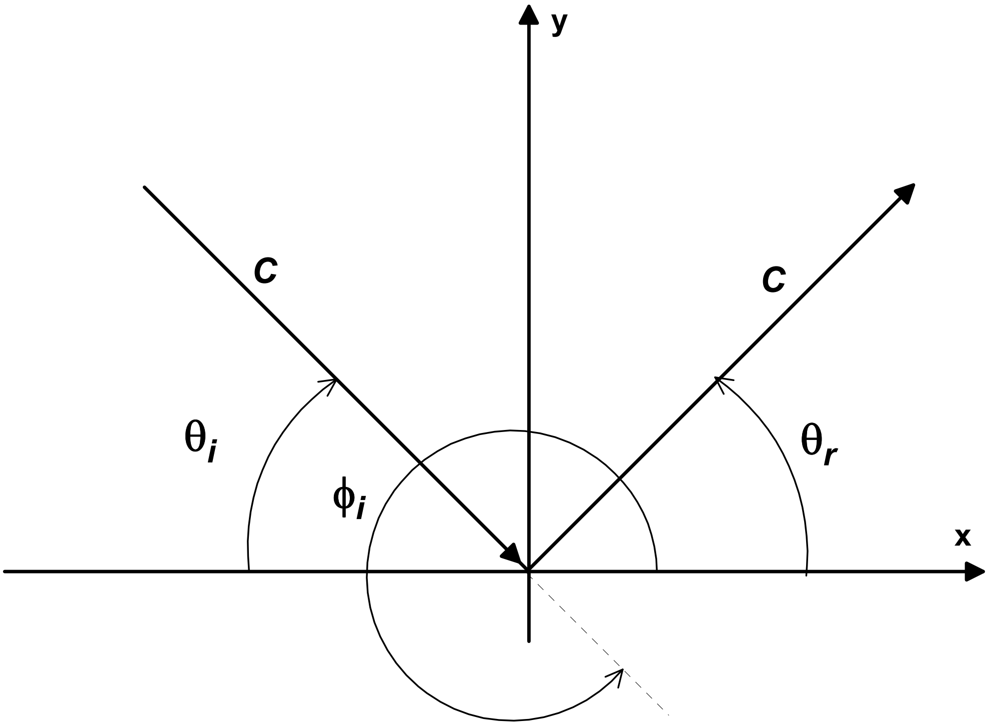


*Fig 5*

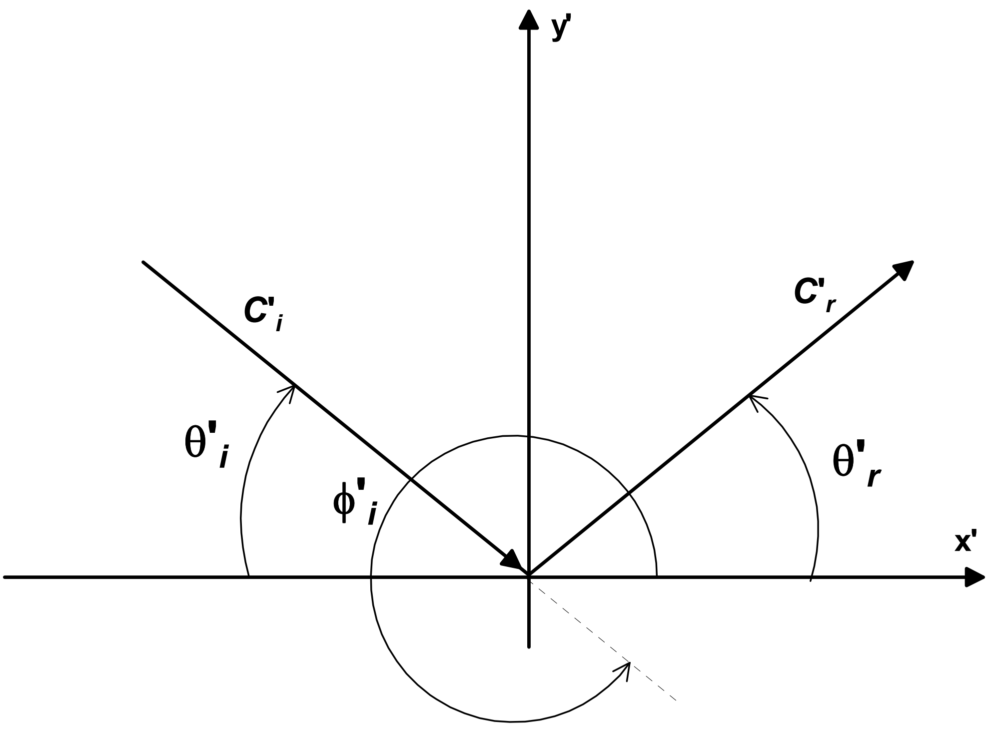


*Fig. 6*

**According to the Lorentz transform, we have:**

$$\cos\left(\phi'_i\right) = \frac{\cos\left(\phi_i\right) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos\left(\phi_i\right)} \qquad \cos\left(\theta'_r\right) = \frac{\cos\left(\theta_r\right) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos\left(\theta_r\right)} \qquad c'_i = c'_r = c$$

$\phi_i = 2\pi - \theta_i$ and $\phi'_i = 2\pi - \theta'_i$, hence $\cos(\phi_i) = \cos(\theta_i)$ and $\cos(\phi'_i) = \cos(\theta'_i)$

Hence: $\cos\left(\theta'_i\right) = \dfrac{\cos\left(\theta_i\right) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos\left(\theta_i\right)}$

Since $\theta_r = \theta_i$, we get: $\cos\left(\theta'_r\right) = \dfrac{\cos\left(\theta_i\right) - \frac{v}{c}}{1 - \frac{v}{c}\cdot\cos\left(\theta_i\right)} = \cos\left(\theta'_i\right)$

Thus $\theta'_r = \theta'_i$

**A light beam incident upon a moving mirror will be reflected at an angle equal to the incident angle.**

**According to the Galilean transform, we have:**

$$\cos\left(\phi'_i\right) = \frac{\cos\left(\phi_i\right) - \frac{v}{c}}{\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos\left(\phi_i\right) + \left(\frac{v}{c}\right)^2}} \qquad c'_i = c\cdot\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos\left(\phi_i\right) + \left(\frac{v}{c}\right)^2}$$

$$\cos\left(\theta_r'\right) = \frac{\cos\left(\theta_r\right) - \frac{v}{c}}{\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos\left(\theta_r\right) + \left(\frac{v}{c}\right)^2}} \qquad c'_r = c\cdot\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos\left(\theta_r\right) + \left(\frac{v}{c}\right)^2}$$

Since $\cos(\phi_i) = \cos(\theta_i)$ and $\cos(\phi'_i) = \cos(\theta'_i)$ we find:

$$\cos\left(\theta'_i\right) = \frac{\cos\left(\theta_i\right) - \frac{v}{c}}{\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos\left(\theta_i\right) + \left(\frac{v}{c}\right)^2}} \qquad c'_i = c\cdot\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos\left(\theta_i\right) + \left(\frac{v}{c}\right)^2}$$

Since $\theta_r = \theta_i$, we get: $\cos\left(\theta'_r\right) = \dfrac{\cos\left(\theta_i\right) - \frac{v}{c}}{\sqrt{1 - 2\cdot\frac{v}{c}\cdot\cos\left(\theta_i\right) + \left(\frac{v}{c}\right)^2}} = \cos\left(\theta'_i\right)$

Thus $\theta'_r = \theta'_i$ and $c'_r = c'_i$.

**A light beam incident upon a moving mirror will be reflected at an angle equal to the incident angle.**

**The speed of a light beam reflected off a moving mirror will be the same as the speed of the incident beam.**