Yet another thought experiment by Ralph Malcolm Rabbidge

Paul B. Andersen

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1 Introduction

Ralph Malcolm Rabbidge, who is posting in the usenet-group sci.physics.relativity under the pseudonym Henry Wilson, has presented a thought experiment. He claims that the predictions of the Special theory of Relativity are logically impossible.

I will show that this is not the case.

Ralph Malcolm Rabbidge's original post

Begin quotation of Ralph Malcolm Rabbidge's description of the though experiment

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We have two identical and perfect rods, one with a clock at each end. The
clocks are e-synched in agreement with BaTh. (although accurate synching
doesn't really matter in this experiment) At a certain instant, the rod will
be caused to move to the right at v.

                      C1________________________C2
        A_________________________B  ->v

The clocks are capable of detecting the exact instant when the ends are
adjacent. C1 will detect A and record the time, C2 will detect B and record
that time.

According to some science fiction theories, in the clock frame, the rod is
shortened. Therefore A will reach C1 before B reaches C2. C1's reading will
be lower than C2's
In the rod frame, the clocks are moving to the left and their rod is
shortened. Therefore C2 will reach B before C1 reaches A. C2's reading will
be lower than that of C1.

Can some knowledgable person please explain this apparent logical
impossibility...
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End quotation =====================

2 Definition of the thought experiment

We have made a slight modification in the thought experiment described by Ralph Malcolm Rabbidge, we have placed clocks at the ends of both rods. This will obviously not affect the readings of clocks C_1 and C_2.

We have two identical rods, both rods have the length L. Each rod has a clock at each end, called A and B, and C_1 and C_2 respectively. The rods are parallel to each other, and are in inertial motion with a speed v relative to each other, see fig. 1. The two pairs of clocks are synchronized in their respective rest frames, and are set such that that clock A and C_1 both shows 0 at the instant when they are transverse to each other.

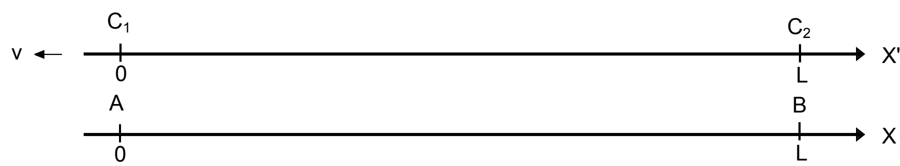

Figure 1: *The frames of reference*

The problem is to determine the order of the events A and C_1 adjacent, and B and C_2 adjacent in the two frames of reference.

3 Calculation of the thought experiment according to the SR

There are two events of interest:
E_1: Clock A and C_1 are adjacent
E_2: Clock B and C_2 are adjacent

The coordinates of these events in the rest frame of clock A and B are:

$$\begin{array}{lll} E_1: & t_A = 0 & x_A = 0 \\ E_2: & t_B & x_B = L \end{array}$$

The coordinates of these events in the rest frame of clock C_1 and C_2 are:

$$\begin{array}{lll} E_1: & t'_{C1} = 0 & x'_{C1} = 0 \\ E_2: & t'_{C2} & x'_{C2} = L \end{array}$$

The Lorentz transform applied on event E_2 gives the following equations to determine t_B and t'_{C2}:

$$x'_{C2} = \gamma\left(x_B + vt_B\right)$$
$$x_B = \gamma\left(x'_{C2} - vt'_{C2}\right)$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Inserting $x_B = L$ and $x'_{C2} = L$ and solving, yields:

$$t_B = -\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

$$t'_{C2} = \frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

4 The length of the rod A-B measured in the rest frame of the rod C_1-$C2$

4.1 The length of the rod at the time $t' = 0$

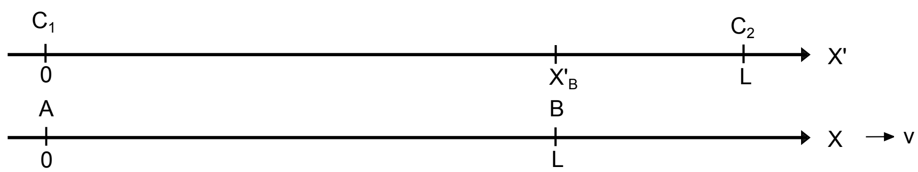

Figure 2: *The rods at the time* $t' = 0$

At the time $t' = 0$ we know that clock A is adjacent to clock C_1. Clock B is at this time at some position x'_B, showing a time t_{B1}. We can then write:

$$t' = \gamma\left(t_{B1} + \frac{xv}{c^2}\right)$$

$$x'_B = \gamma\left(x + vt_{B1}\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We know that $t' = 0$ and $x = L$, so the equations can be written:

$$0 = \gamma\left(t_{B1} + \frac{Lv}{c^2}\right)$$

$$x'_B = \gamma\left(L + vt_{B1}\right)$$

Solving these, yields:

$$t_{B1} = -\frac{Lv}{c^2}$$

$$x'_B = L\sqrt{1 - \frac{v^2}{c^2}}$$

This means that as observed in the rest frame of rod C_1-C_2 at the time when clock A is adjacent to clock C_1, the measured length of rod A-B is contracted to $L\sqrt{1 - \frac{v^2}{c^2}}$ and clock B is reading $-\frac{Lv}{c^2}$.

4.2 The length of the rod at the time $t' = \frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$

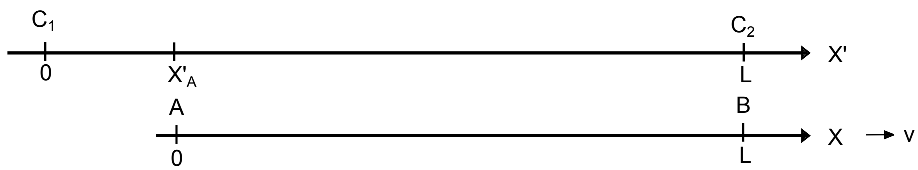

Figure 3: *The rods at the time* $t' = \frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$

At the time $t' = \frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$ we know that clock B is adjacent to clock C_2. Clock A is at this time at some position x'_A, showing a time t_{A1}. We can then write:

$$t' = \gamma\left(t_{A1} + \frac{xv}{c^2}\right)$$
$$x'_A = \gamma\left(x + vt_{A1}\right)$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We know that $t' = \frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$ and $x = 0$, so the equations can be written:

$$\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right) = \gamma\left(t_{A1} + 0\right)$$
$$x'_A = \gamma\left(0 + vt_{A1}\right)$$

Solving these, yields:

$$t_{A1} = \frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)\sqrt{1 - \frac{v^2}{c^2}}$$
$$x'_A = L\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

Note that the measured length of rod A-B is $(L - x'_A) = L\sqrt{1 - \frac{v^2}{c^2}}$.

This means that as observed in the rest frame of rod C_1-C_2 at the time when clock B is adjacent to clock C_2, the measured length of rod A-B is contracted to $L\sqrt{1 - \frac{v^2}{c^2}}$ and clock A is reading $\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)\sqrt{1 - \frac{v^2}{c^2}}$.

5 The length of the rod C_1-C_2 measured in the rest frame of the rod A-B

5.1 The measured length of the rod at the time $t = -\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$

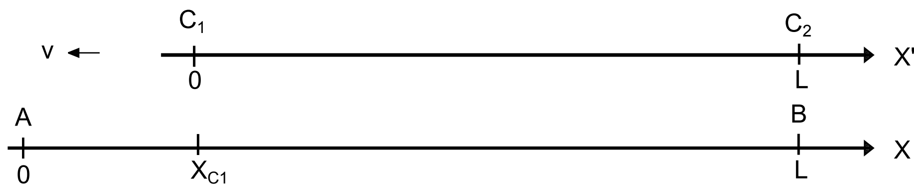

Figure 4: *The rods at the time* $t = -\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$

At the time $t = -\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$ we know that clock C_2 is adjacent to clock B. Clock C_1 is at this time at some position x_{C1}, showing a time t'_{C11}. We can then write:

$$t = \gamma\left(t'_{C11} - \frac{x'v}{c^2}\right)$$
$$x_{C1} = \gamma\left(x' - vt'_{C11}\right)$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We know that $t = -\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$ and $x' = 0$, so the equations can be written:

$$-\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right) = \gamma\left(t'_{C11} - 0\right)$$
$$x_{C1} = \gamma\left(0 - vt'_{C11}\right)$$

Solving these, yields:

$$t'_{C11} = -\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)\sqrt{1 - \frac{v^2}{c^2}}$$
$$x_{C1} = L\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

Note that the measured length of rod C_1-C_2 is $(L - x_{C1}) = L\sqrt{1 - \frac{v^2}{c^2}}$.

This means that as observed in the rest frame of rod A-B at the time when clock C_2 is adjacent to clock B, the measured length of rod C_1-C_2 is contracted to $L\sqrt{1 - \frac{v^2}{c^2}}$ and clock C_1 is reading $-\frac{L}{v}\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)\sqrt{1 - \frac{v^2}{c^2}}$.

5.2 The length of the rod at the time $t = 0$

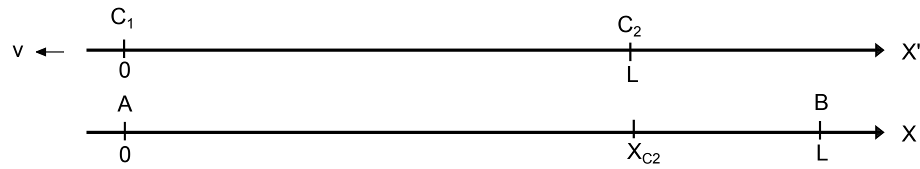

Figure 5: *The rods at the time $t = 0$*

At the time $t = 0$ we know that clock C_1 is adjacent to clock A. Clock C_2 is at this time at some position x_{C2}, showing a time t'_{C21}.

We can then write:

$$t = \gamma \left(t'_{C21} - \frac{x'v}{c^2} \right)$$
$$x_{C2} = \gamma \left(x' - vt'_{C21} \right)$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

We know that $t = 0$ and $x' = L$, so the equations can be written:

$$0 = \gamma \left(t'_{C21} - \frac{Lv}{c^2} \right)$$
$$x_{C2} = \gamma \left(L - vt'_{C21} \right)$$

Solving these, yields:

$$t'_{C21} = \frac{Lv}{c^2}$$
$$x_{C2} = L\sqrt{1 - \frac{v^2}{c^2}}$$

This means that as observed in the rest frame of rod A-B at the time when clock C_1 is adjacent to clock A, the measured length of rod C_1-C_2 is contracted to $L\sqrt{1 - \frac{v^2}{c^2}}$ and clock C_2 is reading $\frac{Lv}{c^2}$.

6 Conclusion

When the clocks are set such that clock A and clock C_1 both shows 0 at the instant when they are adjacent, then clock B shows $-\frac{L}{v}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)$ while clock C_2 shows $+\frac{L}{v}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)$ at the instant when they are adjacent.

So in the rest frame of clocks A and B, the event A and $C1$ adjacent happens after the event B and $C2$ adjacent, while in the rest frame of clocks $C1$ and $C2$, the event A and $C1$ adjacent happens before the event B and $C2$ adjacent.

This is an example of "relativity of simultaneity", and it is not clear what Ralph Malcolm Rabbidge finds logically impossible. However, in a posting he wrote:

> *"Is the READING of C1 larger or smaller than that of C2? Those are pure NUMBERS and obviously must be the same in ALL frames. According to SR, C1's 'number' should be both larger and smaller than C2's"*

This strange statement indicates that Ralph Malcolm Rabbidge is very confused about what relativity of simultaneity is.