Yet another thought experiment by Ralph Malcolm Rabbidge

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1 Introduction

Ralph Malcolm Rabbidge, who is posting in the usenet-group sci.physics.relativity under the pseudonym Henry Wilson, has presented a thought experiment for which he claims that the Special Theory of Relativity (hereafter called SR) will give contradictory predictions depending on in which frame of reference the calculations of the predictions are made.

I will show that this is not the case.

Ralph Malcolm Rabbidge's original post

from where I quote:

> Two clocks A and B are connected by a calibrated rod, length 2L. At the mid point of the rod, P, is observer, Os, holding a light source and identical clock, Cs. Observer, Om, attached to the rod synchronizes the two clocks using light signals from that source.
>
> $$A\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_P\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_B...Om$$
>
> $$Os/Cs/S$$
>
> Next, Os performs a series of experiments whereby he moves from left of A to right of B, at exactly the same speed according to the rod calibrations and his own clock. In the process, he adjusts the reading and rate of his clock so that its reading always coincides with that of A and B when either one is adjacent.
>
> That having been achieved, during the next run the source emits a flash of light in both directions when precisely adjacent to point P. Cs is set to zero at that instant. A and B subsequently record their readings when the flash arrives.
>
> According to Einstein's theory, in the frame of Om, the flash moves at c irrespective of the speed of the source and so both A and B, having been synched with such light, must read the SAME time when it arrives at their respective locations.
>
> In the frame of Os, however, both A and B are moving to the left. (Os will have already calculated their speed as v, in his frame, from the rod calibrations and his own clock, Cs...but the actual value of v is not important).
>
> Since light travels at c in his frame, Os is able to CALCULATE that the flashes arrive at A and B when his clock reads L/(c-v) and L/(c+v) respectively.
>
> Since his clock readings are known to be always identical to those of A and B he knows that those clocks must record DIFFERENT readings when the flashes arrive.
>
> So in one frame, the recorded readings of A and B must the same, in the other frame, they must be different.

End quotation ————————————————————————————-

2 The thought experiment

2.1 Comments to R.M. Rabbidge's description of the thought experiment

Note that the only question is what SR predicts the two clocks A and B will read when they are hit by the flash of light. R. M. Rabbidge claims that observer Om will find that SR predicts they read the same, while observer Os will find that SR predicts the readings will be different. If that is correct, SR is inconsistent.

Note also that according to the description above, the clock C_s can't have the same intrinsic rate as the clocks A and B; it must run faster by the factor $\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$. This is however utterly irrelevant, because this clock is never used for the timing of anything. Clock C_s couldn't affect the reading of clock A or B, even if it were running backwards.

2.2 Definition of the thought experiment

In an inertial frame of reference K there are two synchronized, stationary clocks, A and B. The distance between the clocks is $2L$. The point P is midway between the clocks.

An observer Os is moving along a line between the clocks, in the direction from A to B. The speed is v as measured in K. The observer has a light source which emits a flash of light at the instant when it is co-located with the point P. We will call the rest frame of observer Os K'.

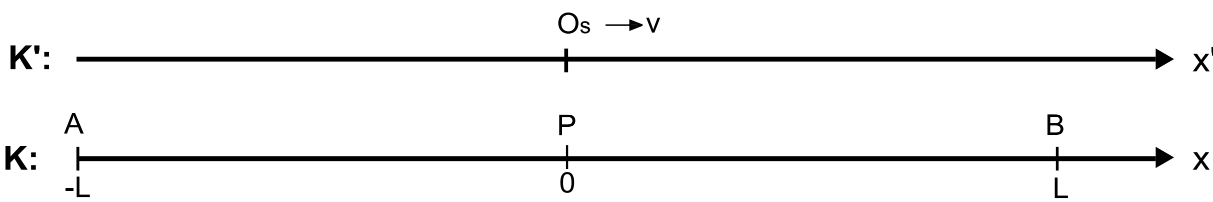

Figure 1: *The frames of reference*

There are three events of interest:

- Let E_0 be the event that a light pulse is emitted by Os as it is co-located with the point P. Let the coordinate time of K be zero at this event.
- Let E_1 be the event that clock A is hit by the light pulse. Let's call the reading of A at this event t_A
- Let E_2 be the event that clock B is hit by the light pulse. Let's call the reading of B at this event t_B

2.3 Calculation of t_A and t_B in the frame of reference K

Let us assume that the two clocks are set equal to the coordinate time of their rest frame K. This means that both clocks are reading zero at the event E_0, and we get:

$$t_A = \frac{L}{c} \tag{1}$$

$$t_B = \frac{L}{c} \tag{2}$$

2.4 Calculation of t_A and t_B in the frame of reference K'

Let us first find what the clocks A and B are showing when the light pulse is emitted at P. Let's call these readings t_{A0} and t_{B0} respectively. Let's call the position of the clocks A and B in K' at this time x'_{A0} and x'_{B0} respectively.

The Lorentz transform will then give us these equations:

$$-L = \frac{x'_{A0} + v \cdot 0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

$$t_{A0} = \frac{0 + \frac{x'_{A0} \cdot v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

$$L = \frac{x'_{B0} + v \cdot 0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5}$$

$$t_{B0} = \frac{0 + \frac{x'_{B0} \cdot v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{6}$$

Solving these equations yields:

$$t_{A0} = -\frac{Lv}{c^2} \tag{7}$$

$$x'_{A0} = -L\sqrt{1 - \frac{v^2}{c^2}} \tag{8}$$

$$t_{B0} = \frac{Lv}{c^2} \tag{9}$$

$$x'_{B0} = L\sqrt{1 - \frac{v^2}{c^2}} \tag{10}$$

Since clock A is moving away from Os, we get the following equation to determine the transit time $\Delta t'_A$ of the flash of light which hits A, as measured in frame K':

$$\Delta t'_A \cdot c = L\sqrt{1 - \frac{v^2}{c^2}} + \Delta t'_A \cdot v \tag{11}$$

$$\Delta t'_A = \frac{L\sqrt{1 - \frac{v^2}{c^2}}}{c - v} \tag{12}$$

Since clock B is moving towards Os, we get the following equation to determine the transit time $\Delta t'_B$ of the flash of light which hits B, as measured in frame K':

$$\Delta t'_B \cdot c + \Delta t'_B \cdot v = L\sqrt{1 - \frac{v^2}{c^2}} \tag{13}$$

$$\Delta t'_B = \frac{L\sqrt{1 - \frac{v^2}{c^2}}}{c + v} \tag{14}$$

Since the clocks A and B are moving with speed v in frame K', they are 'running slow', and will advance an amount:

$$\Delta t_A = \Delta t'_A\sqrt{1 - \frac{v^2}{c^2}} = \frac{L\left(1 - \frac{v^2}{c^2}\right)}{c - v} = \frac{L}{c}\left(1 + \frac{v}{c}\right) \tag{15}$$

$$\Delta t_B = \Delta t'_B\sqrt{1 - \frac{v^2}{c^2}} = \frac{L\left(1 - \frac{v^2}{c^2}\right)}{c + v} = \frac{L}{c}\left(1 - \frac{v}{c}\right) \tag{16}$$

The clock A will at the event E_1 show:

$$t_A = t_{A0} + \Delta t_A = -\frac{Lv}{c^2} + \frac{L}{c}\left(1 + \frac{v}{c}\right) = \frac{L}{c} \tag{17}$$

and the clock B will at the event E_2 show:

$$t_B = t_{B0} + \Delta t_B = \frac{Lv}{c^2} + \frac{L}{c}\left(1 - \frac{v}{c}\right) = \frac{L}{c} \tag{18}$$

3 Conclusion

We have shown that when calculated in the rest frame of clocks A and B, SR predicts that both clocks show the same, $t_A = t_B = \frac{L}{c}$.

We have shown that when calculated in the rest frame of Os, SR predicts that both clocks show the same, $t_A = t_B = \frac{L}{c}$.

Ralph Malcolm Rabbidge's claim that the Special Theory of Relativity will give contradictory predictions depending on in which frame of reference the calculations of the predictions are made,
is therefore false.