Yet another thought experiment by Ralph Malcolm Rabbidge

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1 Introduction

Ralph Malcolm Rabbidge, who is posting in the usenet-group sci.physics.relativity under the pseudonym Henry Wilson, has presented a thought experiment for which he claims that the Special Theory of Relativity (hereafter called SR) will give contradictory predictions depending on in which frame of reference the calculations of the predictions are made.

I will show that this is not the case.

Ralph Malcolm Rabbidge's original post

2 The thought experiment

2.1 Definition of the thought experiment

In an inertial frame of reference K there are three synchronized, stationary clocks, C_1, C_2 and C_3. C_1 and C_2 are co-located at the position $x = 0$, while C_3 is at the position $x = L$.

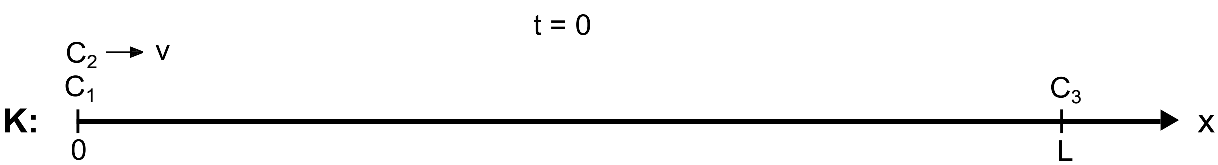

Figure 1: *Frame K at $t = 0$*

When the clocks all show 0, clock C_2 is instantly set in motion towards clock C_3 with the speed v.

Let E_0 be the event that clock C_2 is at $x = 0$, showing 0, and starts moving.

Let E_1 be the event that clock C_2 is at $x = L$ and is co-located with C_3.

Let E_2 be the event that clock C_3 is at $x = L$ showing 0.

The problem is to find what C_2 and C_3 show when they are co-located at event E_1.

Let T_{C2} be what clock C_2 shows when it is co-located with C_3 at event E_1.

Let T_{C3} be what clock C_3 shows when it is co-located with C_2 at event E_1.

2.2 Calculation of T_{C2} and T_{C3}

Let K be the rest frame of clock C_1 and C_3, and let K' be the rest frame of clock C_2.
Let us choose coordinate systems as shown in fig.2.

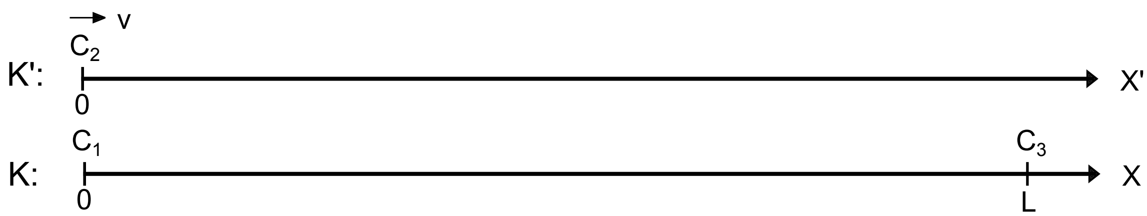

Figure 2: *The frames of reference K and K'*

2.2.1 Calculation of T_{C2} and T_{C3} in the rest frame of clocks C_1 and C_3

Clock C_2, which is moving the distance L at the speed v, will reach clock C_3 when the latter shows:

$$T_{C3} = \frac{L}{v}$$

Clock C_2 is showing zero at event E_0, and is running slow by the factor $\sqrt{1-\frac{v^2}{c^2}}$ as observed in frame K. At event E_1 clock C_2 will thus show:

$$T_{C2} = \frac{L}{v}\sqrt{1-\frac{v^2}{c^2}}$$

A more proper way to find T_{C2} is however to transform the coordinates of event E_1 to frame K', the rest frame of clock C_2.

The coordinates of the event E_1 in frame K are: $x_1 = L$, $t_1 = \frac{L}{v}$.

The coordinates of the event E_1 transformed to frame K' are:

$$x_1' = \frac{x_1 - vt_1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L - v\frac{L}{v}}{\sqrt{1-\frac{v^2}{c^2}}} = 0$$

$$t_1' = \frac{t_1 - \frac{vx_1}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{L}{v} - \frac{vL}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{L}{v}\sqrt{1-\frac{v^2}{c^2}}$$

Since clock C_2 is stationary in frame K', t_1' is a proper time, so:

$$T_{C2} = t_1' = \frac{L}{v}\sqrt{1-\frac{v^2}{c^2}}$$

2.2.2 Calculation of T_{C2} and T_{C3} in the rest frame of clock C_2

The coordinates of the event E_2 in frame K are: $x_2 = L$, $t_2 = 0$

The coordinates of the event E_2 transformed to K' are:

$$x_2' = \frac{x_2 - vt_2}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_2' = \frac{t_2 - \frac{vx_2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = -\frac{L}{v}\frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Clock C_3 starts at the time t_2' and is moving the distance x_2' with the speed v, so when clock C_3 meets clock C_2, the latter will show:

$$T_{C2} = t_2' + \frac{x_2'}{v} = -\frac{L}{v}\frac{\frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} + \frac{\frac{L}{\sqrt{1-\frac{v^2}{c^2}}}}{v} = \frac{L}{v}\sqrt{1 - \frac{v^2}{c^2}}$$

Clock C_3 will start moving (showing 0) from the position x_2', so it will move the distance x_2' with the speed v. As observed in frame K', clock C_3 will run slow by the factor $\sqrt{1 - \frac{v^2}{c^2}}$.

When clock C_3 meets clock C_2, the former will thus show:

$$T_{C3} = \frac{x_2'}{v}\sqrt{1 - \frac{v^2}{c^2}} = \left(\frac{\frac{L}{\sqrt{1-\frac{v^2}{c^2}}}}{v}\right)\sqrt{1 - \frac{v^2}{c^2}} = \frac{L}{v}$$

3 Conclusion

We have shown that when calculated in the rest frame of clocks C_1 and C_3, SR predicts that $T_{C3} = \frac{L}{v}$ and $T_{C2} = \frac{L}{v}\sqrt{1 - \frac{v^2}{c^2}}$.

We have shown that when calculated in the rest frame of clock C_2, SR predicts that $T_{C3} = \frac{L}{v}$ and $T_{C2} = \frac{L}{v}\sqrt{1 - \frac{v^2}{c^2}}$.

Ralph Malcolm Rabbidge's claim that the Special Theory of Relativity will give contradictory predictions depending on in which frame of reference the calculations of the predictions are made, is therefore false.