# Moving Ball

Paul B. Andersen

February 13, 2013

## 1 Transformation of the velocity of a moving ball

In the inertial frame of reference $K$, a ball is moving along the y-axis with the speed $u$.

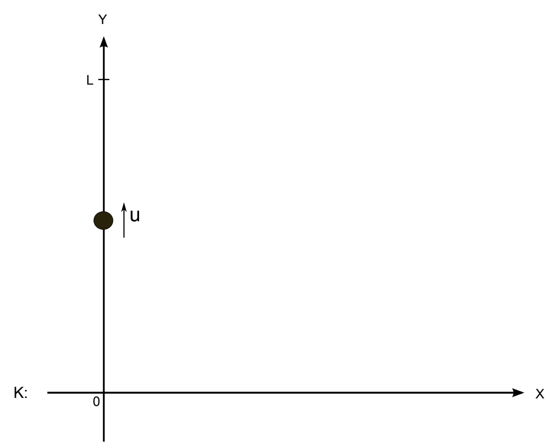


Figure 1: *The frame of reference K*

A second frame of reference $K'$ is moving along the negative x-axis of $K$.

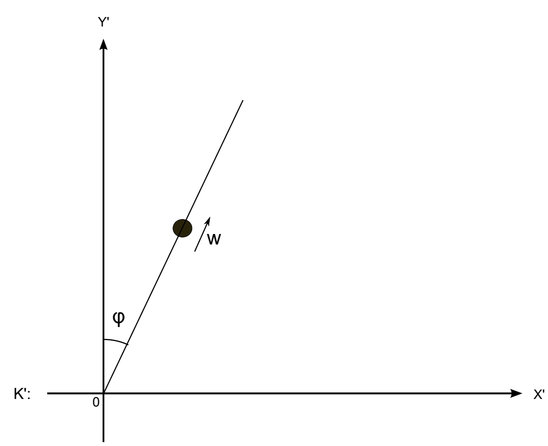


Figure 2: *The frame of reference K′*

What will the speed of and direction of the ball be in $K'$ ?

Let $E_0$ be the event when the ball is at the origin of $K$, and let $E_1$ be the event when the ball is at the y-axis a distance L from the origin in $K$.

The coordinates of event $E_0$ in $K$ are: $t_0 = 0$, $x_0 = 0$, $y_0 = 0$
The coordinates of event $E_1$ in $K$ are: $t_1 = \frac{L}{u}$, $x_1 = 0$, $y_1 = L$

## 1.1 Transformation according to SR

Transformed to $K'$ the coordinates of event $E_0$ are: $t'_0 = 0$, $x'_0 = 0$, $y'_0 = 0$
Transformed to $K'$ the coordinates of event $E_1$ are:

$$t'_1 = \frac{\left(t_1 + \frac{vx_1}{c^2}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{\frac{L}{u}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$x'_1 = \frac{(x_1 + vt_1)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \frac{\frac{v}{u}L}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$y'_1 = y_1 = L$$

The speed of the ball in $K'$ will be:

$$w = \frac{\sqrt{(x'_1 - x'_0)^2 + (y'_1 - y'_0)^2}}{(t'_1 - t'_0)} = \frac{\sqrt{\left(\frac{\frac{v}{u}L}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}\right)^2 + L^2}}{\frac{\frac{L}{u}}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}} = \sqrt{u^2 + v^2 - \frac{u^2v^2}{c^2}}$$

Note that $w \to c$ when $u- > c$

The angle of the velocity in $K'$ will be $\varphi$ from vertical:

$$\sin\varphi = \frac{(x'_1 - x'_0)}{\sqrt{(x'_1 - x'_0)^2 + (y'_1 - y'_0)^2}} = \frac{\frac{\frac{v}{u}L}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}}{\sqrt{\left(\frac{\frac{v}{u}L}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}}\right)^2 + L^2}} = \frac{1}{\sqrt{1 + \frac{u^2}{v^2} - \frac{u^2}{c^2}}}$$

$$\varphi = \arcsin\left(\frac{1}{\sqrt{1 + \frac{u^2}{v^2} - \frac{u^2}{c^2}}}\right)$$

Note that $\varphi \to \arcsin\left(\frac{v}{c}\right)$ when $u \to c$

## 1.2 Transformation according to Galilean relativity

Transformed to $K'$ the coordinates of event $E_0$ are: $t'_0 = 0$, $x'_0 = 0$, $y'_0 = 0$
Transformed to $K'$ the coordinates of event $E_1$ are:

$$t'_1 = t_1 = \frac{L}{u}$$

$$x'_1 = x_1 + vt_1 = \frac{v}{u}L$$

$$y'_1 = y_1 = L$$

The speed of the ball in $K'$ will be:

$$w = \frac{\sqrt{\left(x_1' - x_0'\right)^2 + \left(y_1' - y_0'\right)^2}}{(t_1' - t_0')} = \frac{\sqrt{\left(\frac{v}{u}L\right)^2 + L^2}}{\frac{L}{u}} = \sqrt{u^2 + v^2}$$

The angle of the velocity in $K'$ will be $\varphi$ from vertical:

$$\tan\varphi = \frac{(x_1' - x_0')}{(y_1' - y_0')} = \frac{\frac{v}{u}L}{L} = \frac{v}{u}$$

$$\varphi = \arctan\left(\frac{v}{u}\right)$$

Note that $\varphi \to \arctan\left(\frac{v}{c}\right)$ when $u \to c$