Origin of the Lorentz Transformation

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1 Introduction

If you have a reasonable knowledge of physics, you don't have to read the following. You will know it.

The Lorentz Transformation was introduced by H. A. Lorentz in 1904 in the paper: *Electromagnetic phenomena in a system moving with any velocity less than that of light*

Lorentz: Electromagnetic phenomena . .

2 Finding the Lorentz transformation in Lorentz's paper

For a reader who is not very skilled in mathematics, it may not be obvious that the Lorentz transformation is defined in that paper. What Lorentz does, is to transform Maxwell's equations from a frame of reference which is stationary in the ether, to a frame of reference which is moving through the ether, using the Galilean transform. He then, as he puts it, "transform these formulae further by a change of variables". He then ends up with Maxwell's equations on an invariant form. These two transformations together constitute what now is known as the Lorentz transformation.

In §3, at the bottom of page 13, he writes: *"I shall now suppose that the system as a whole moves in the direction of x with a constant velocity v"*.

The Galilean transformation yields:

$$x'' = x - vt \tag{1}$$

where x and t are coordinates in the ether frame, and x'' is a coordinate in the Galilean transformed moving frame. All other coordinates (y, z and t) are unaffected by the Galilean transformation.

In §4, at the bottom of page 14, he writes:

> § 4. We shall further transform these formulæ by a change of variables. Putting
>
> $$\frac{c^2}{c^2 - v^2} = \beta^2, \quad . \quad . \quad . \quad . \quad (3)$$
>
> and understanding by l another numerical quantity, to be determined further on, I take as new independent variables
>
> $$x' = \beta l x, \; y' = l y, \; z' = l z, \quad . \quad . \quad . \quad (4)$$
>
> $$t' = \frac{l}{\beta} t - \beta l \frac{v}{c^2} x, \quad . \quad . \quad . \quad . \quad (5)$$

Note that $\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. The x is the x-coordinate which is transformed to the moving frame using the Galilean transform, that is the coordinate we denoted x'' in equation (1). The l is a scaling factor which later was determined to be 1.

So this "change of variables" transformation can be written:

$$\begin{aligned}
x' &= \frac{x''}{\sqrt{1 - \frac{v^2}{c^2}}} \\
y' &= y \\
z' &= z \\
t' &= \sqrt{1 - \frac{v^2}{c^2}}\, t - \frac{\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\, x''
\end{aligned}$$

If we substitute for x'' using equation (1), we get:

$$\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{aligned}$$

which is the Lorentz transformation as we are used to see it.

3 The Lorentz Ether Theory (LET)

By Lorentz Ether Theory we mean the theory defined by the Lorentz transformation, with the interpretation that the unprimed coordinates belong to a frame of reference stationary in the ether, and the primed coordinates belong to a frame of reference which is moving through the ether.

However, since the Lorentz transformation forms a group, it can be used to transform coordinates of events between any arbitrary pair of frames of reference.

Let $K(x,t)$ be the ether frame, let $K'(x',t')$ be a frame of reference which is moving at the speed v through the ether, and let $K''(x'', t'')$ be a frame of reference which is moving at the speed u through the ether.

We then have

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and

$$x'' = \frac{x - ut}{\sqrt{1 - \frac{u^2}{c^2}}}, \quad t'' = \frac{t - \frac{u}{c^2}x}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Combining these, yields:

$$x'' = \frac{x' - wt'}{\sqrt{1 - \frac{w^2}{c^2}}}, \quad t'' = \frac{t' - \frac{w}{c^2}x'}{\sqrt{1 - \frac{w^2}{c^2}}}$$

where $w = \dfrac{v - u}{1 - \frac{vu}{c^2}}$ is the relative speed between K' and K''.

Since any arbitrary frame of reference must be assumed to move through the ether, this property of the Lorentz transform make the state of motion of the ether unmeasurable. The w above can be measured, the v and u can not.

Lorentz called the t' coordinate 'local time', as opposed to the t coordinate which was the 'absolute time' inherited from Newton. But note that this 'local time' is what it shown by local clocks, and it is the 'local time' that can be measured. The 'absolute time' is unobservable.

Since LET is defined by the Lorentz transformation, and this transformation also applies in the Special Theory of Relativity (SR), LET and SR will always predict the same for any experiment.