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Is the Lorentz transform inconsistent?

Ufuk Denizyar claims in his paper: http://www.geocities.com/udenizyar/lt3.pdf that the Lorentz transform is inconsistent. Below I will calculate his "Experiment 2" and show that this is not the case. I have done one small simplification: I have set his $t_0 = 0$. This has no consequence for the result.

The scenario

Let A, B and C be three observers who at some time are co-located. Let C be stationary in an inertial frame hereafter called "frame C". Let A be moving at the speed v to the left, while B is moving at the speed v to the right. Let each observer have a clock, called clock A, B and C respectively. Let these clocks be set to zero at the instant when the observers are co-located.

When clock C is showing the time δ, C emits a radio signal.
What are clock A and B showing when they receive the radio signal?

Calculation in frame C

The speed of light is c in frame C.

Let the coordinates of A in frame C when he receives the signal be x_{CA} and t_{CA}, while the coordinates of B in frame C are x_{CB} and t_{CB} when he receives the signal.

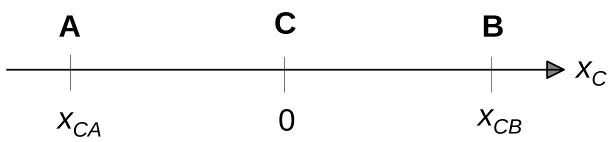

Fig. 1
Positions in frame C at the time when the signals are received

A will be at the position $x_{CA} = -vt_{CA}$ when the signal is received. The transit time of the signal is $(t_{CA} - \delta)$, and it is moving with the speed c, so $x_{CA} = -(t_{CA} - \delta)c$
Solving these equations yields the coordinates:

$$x_{CA} = -\frac{cv}{c-v}\delta \qquad t_{CA} = \frac{c}{c-v}\delta$$

Equivalently we find the coordinates of B to be:

$$x_{CB} = \frac{cv}{c-v}\delta \qquad t_{CB} = \frac{c}{c-v}\delta$$

Let "frame A" and "frame B" be A's and B's rest frames respectively.
Let the coordinates of A in frame A when he receives the signal be x_A and t_A, while the coordinates of B in frame B are x_B and t_B when he receives the signal.
To find these coordinates we must apply the Lorentz transform:

$$x_A = \frac{x_{CA} + v \cdot t_{CA}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{-\frac{cv}{c-v}\delta + v \cdot \frac{c}{c-v}\delta}{\sqrt{1-\frac{v^2}{c^2}}} = 0 \quad \text{(A is at origo of frame A)}$$

$$t_A = \frac{t_{CA} + \frac{v \cdot x_{CA}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{c}{c-v}\delta + \frac{v \cdot \left(-\frac{cv}{c-v}\delta\right)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{c}{c-v}\delta \cdot \left(1-\frac{v^2}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \cdot \delta$$

And:

$$x_B = \frac{x_{CB} - v \cdot t_{CB}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{cv}{c-v}\delta - v\frac{c}{c-v}\delta}{\sqrt{1-\frac{v^2}{c^2}}} = 0 \quad \text{(B is at origo of frame B)}$$

$$t_B = \frac{t_{CB} - \frac{v \cdot x_{CA}}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{c}{c-v}\delta - \frac{v \cdot \frac{cv}{c-v}\delta}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\frac{c}{c-v}\delta \cdot \left(1-\frac{v^2}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \cdot \delta$$

So both clock A and clock B will show $\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \cdot \delta$ when they receive the signal.

Calculation in frame A

The speed of light is c in frame A.

If it is possible to calculate this time to be different by doing the calculation in a different frame of reference than the above, then the Lorentz transform is inconsistent.
So let us do the calculations in frame A and see.

C's speed in frame A is v, while B's speed in frame A is $v_B = \frac{2v}{1+\frac{v^2}{c^2}}$

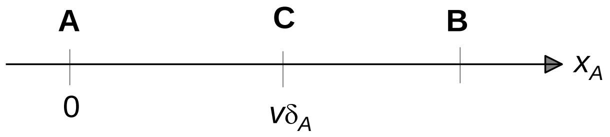

Fig.2
Positions in frame A at the time when the signals are emitted

Let δ_A be the time in frame A when the light is emitted from C. At this time, C is at position $v\delta_A$ in frame A. We know that clock C then is showing δ.
According to the Lorentz transform, we have:

$$\delta = \frac{\delta_A - \frac{v(v\delta_A)}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{1-\frac{v^2}{c^2}} \cdot \delta_A$$

$$\delta_A = \frac{\delta}{\sqrt{1-\frac{v^2}{c^2}}}$$

The signal will reach A at the time:

$$t_A = \delta_A + \frac{v\delta_A}{c} = \left(1+\frac{v}{c}\right)\frac{\delta}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} \cdot \delta$$

This is the same result as above. No inconsistency so far.

But what will clock B show when the signal is received?

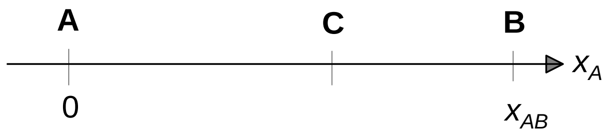

Fig.3
Positions in frame A at the time when B receives the signal

Let the coordinates of B in frame A be x_{AB} and t_{AB} when he receives the signal.

C is at the position $v\delta_A$ when the signal is transmitted, and B will be at $x_{AB} = v_B t_{AB}$ when the signal is received. The travelled distance is thus $\left(x_{AB} - v\delta_A\right)$, the transit time of the signal is $\left(t_{AB} - \delta_A\right)$ and it is moving with the sped c. So $x_{AB} - v\delta_A = \left(t_{AB} - \delta_A\right)c$
Solving these equations yields the coordinates:

$$x_{AB} = \frac{1-\frac{v}{c}}{1-\frac{v_B}{c}} \cdot v_A \delta_A \qquad t_{AB} = \frac{1-\frac{v}{c}}{1-\frac{v_B}{c}} \cdot \delta_A$$

To find what clock B shows when the signal is received, we must transform these coordinates to frame B.

$$x_B = \frac{x_{AB} - v_B \cdot t_{AB}}{\sqrt{1-\frac{v_B^2}{c^2}}} = \frac{v_B \cdot t_{AB} - v_B \cdot t_{AB}}{\sqrt{1-\frac{v_B^2}{c^2}}} = 0 \quad \text{(B is still at origo of frame B)}$$

$$t_B = \frac{t_{AB} - \frac{v_B \cdot x_{AB}}{c^2}}{\sqrt{1-\frac{v_B^2}{c^2}}} = \frac{\left(\frac{1-\frac{v}{c}}{1-\frac{v_B}{c}}\right) \cdot \delta_A - \left(\frac{1-\frac{v}{c}}{1-\frac{v_B}{c}}\right)\left(\frac{v_B^2}{c^2}\right) \cdot \delta_A}{\sqrt{1-\frac{v_B^2}{c^2}}} = \left(\frac{1-\frac{v}{c}}{1-\frac{v_B}{c}}\right)\sqrt{1-\frac{v_B^2}{c^2}} \cdot \delta_A = \left(1-\frac{v}{c}\right)\sqrt{\frac{1+\frac{v_B}{c}}{1-\frac{v_B}{c}}} \cdot \delta_A$$

Inserting $v_B = \frac{2v}{1+\frac{v^2}{c^2}}$ and $\delta_A = \frac{\delta}{\sqrt{1-\frac{v^2}{c^2}}}$ yields:

$$t_B = \left(1-\frac{v}{c}\right)\left(\frac{1+\frac{v}{c}}{1-\frac{v}{c}}\right) \cdot \frac{\delta}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} .\delta = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}} .\delta$$

This is the same as above. No inconsistency.