Koobee Wublee and the Lorentz transform

Paul B. Andersen

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Introduction

The person who is posting in the usenet group sci.physics.relativity under the pseudonym Koobee Wublee has written the following about the Lorentz transform:

Koobee Wublee's text

Koobee Wublee's text with my comments

In the following I will quote from Koobee Wublee's text and comment it.

Quotation from Koobee Wublee's text begins:

> Yes, this is all in the Lorentz transform. Say the twins are A and B. Since any transform requires two observers and the mutually observed. It is necessary to bring in another party, C, mutually observed by A and B. Thus, the time transformation of the Lorentz transform becomes the following. <shrug>
>
> $** \quad dt_{AC} = (dt_{BC} - < v_{BA} > \cdot < ds_{BC} > / c^2) / \sqrt{1 - v_{BA}^2 / c^2}$
>
> And
>
> $** \quad dt_{BC} = (dt_{AC} - < v_{AB} > \cdot < ds_{AC} > / c^2) / \sqrt{1 - v_{AB}^2 / c^2}$, reciprocal
>
> Where
>
> $** \quad dt_{AC}$ = local clock tick rate at C as observed by A
> $** \quad dt_{BC}$ = local clock tick rate at C as observed by B
> $** \quad < v_{AB} >$ = velocity of B as observed by A
> $** \quad < v_{BA} >$ = velocity of A as observed by B
> $** \quad < s_{AC} >$ = position vector of C as observed by A
> $** \quad < s_{BC} >$ = position vector of C as observed by B
> $** \quad < ... > \cdot < ... >$ = dot/inner product of two vectors
>
> The above equations can be written into the following showing how local clock tick rate at C is observed by A and B. <shrug>
>
> $** \quad dt_{AC} = dt_{BC} (1 - < v_{BA} > \cdot < v_{BC} > / c^2) / \sqrt{1 - v_{BA}^2 / c^2}$
>
> And
>
> $** \quad dt_{BC} = dt_{AC} (1 - < v_{AB} > \cdot < v_{AC} > / c^2) / \sqrt{1 - v_{AB}^2 / c^2}$, reciprocal
>
> Where
>
> $** \quad < v_{AC} >= d < s_{AC} > / dt_{AC}$
> $** \quad < v_{BC} >= d < s_{BC} > / dt_{BC}$

Quotation ends

My comment:

Everything is OK so far. I will however define the scenario more precisely.

Given two frames of reference A and B with their x-axes aligned. The origin of A is moving with the speed v along the positive x-axis of B. The origins are aligned when both the temporal coordinates are zero.

An object C is at any time somewhere on the x axes of A and B.

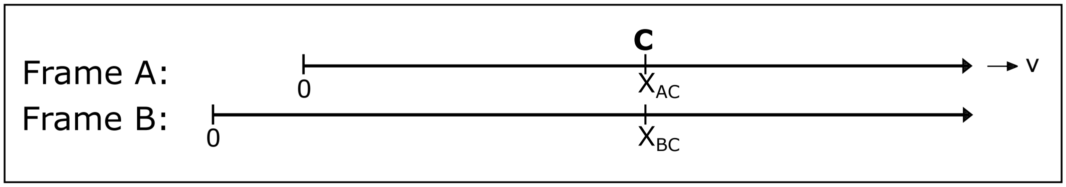

Figure 1: *The frames of reference*

An arbitrary event on C's worldline has the coordinates (t_{AC}, x_{AC}) in frame A and (t_{BC}, x_{BC}) in frame B.

The Lorentz transform:
The coordinates of the event with coordinates (t_{BC}, x_{BC}) in frame B transform to (t_{AC}, x_{AC}) in frame A.

$$t_{AC} = \frac{t_{BC} - \frac{v}{c^2} \cdot x_{BC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \qquad x_{AC} = \frac{x_{BC} - v \cdot t_{BC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{1}$$

The inverse Lorentz transform:
The coordinates of the event with coordinates (t_{AC}, x_{AC}) in frame A transform to (t_{BC}, x_{BC}) in frame B.

$$t_{BC} = \frac{t_{AC} + \frac{v}{c^2} \cdot x_{AC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \qquad x_{BC} = \frac{x_{AC} + v \cdot t_{AC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{2}$$

assuming that v is constant and that C is stationary in either A or B:

$$\frac{\mathrm{d}t_{AC}}{\mathrm{d}t_{BC}} = \frac{\left(1 - \frac{v}{c^2} \cdot \frac{\mathrm{d}x_{BC}}{\mathrm{d}t_{BC}}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{3}$$

$$\frac{\mathrm{d}t_{BC}}{\mathrm{d}t_{AC}} = \frac{\left(1 + \frac{v}{c^2} \cdot \frac{\mathrm{d}x_{AC}}{\mathrm{d}t_{AC}}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{4}$$

or:

$$\mathrm{d}t_{AC} = \frac{\mathrm{d}t_{BC} - \frac{v}{c^2} \cdot \mathrm{d}x_{BC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{5}$$

$$\mathrm{d}t_{BC} = \frac{\mathrm{d}t_{AC} + \frac{v}{c^2} \cdot \mathrm{d}x_{AC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{6}$$

Note that these are the same equations as in the quotation from Koobee Wublee's text above when $ds_{BC} = \mathrm{d}x_{BC}$, $ds_{AC} = \mathrm{d}x_{AC}$, $v_{BA} = v$, $v_{AB} = -v$, $v_{AC} = \frac{\mathrm{d}x_{AC}}{\mathrm{d}t_{AC}}$ and $v_{BC} = \frac{\mathrm{d}x_{BC}}{\mathrm{d}t_{BC}}$.

Quotation from Koobee Wublee's text continues:

> Now, demanding C to be a partner of B, effectively, B and C share the same frame of reference. The above equations simplify into the following. <shrug>
>
> ** $dt_{AB} = dt_{BB} / \sqrt{1 - v_{BA}^2/c^2}$, main
>
> And
>
> ** $dt_{BB} = dt_{AB} \sqrt{1 - v_{AB}^2/c^2}$, reciprocal
>
> Where
>
> ** $dt_{AC} = dt_{AB}$
> ** $dt_{BC} = dt_{BB}$
> ** $< v_{BC} >=< v_{BB} >= 0$
> ** $< v_{AB} > * < v_{AC} >=< v_{AB} > * < v_{AB} >= v_{AB}^2$
>
> The reciprocal agrees with the main equation. Both equations definitely say the local clock tick rate at B is observed to be slow down by A.

Quotation ends

My comment:

The statements "B and C share the same frame of reference" and "$v_{BC} = 0$" can only be interpreted as a convoluted way of stating that C is stationary in frame B.

If C is stationary in frame B we have $\frac{\mathrm{d}x_{AC}}{\mathrm{d}t_{AC}} = -v$ and $\frac{\mathrm{d}x_{BC}}{\mathrm{d}t_{BC}} = 0$. Inserting these in equations (3) and (4) yields:

$$\frac{\mathrm{d}t_{AC}}{\mathrm{d}t_{BC}} = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{7}$$

$$\frac{\mathrm{d}t_{BC}}{\mathrm{d}t_{AC}} = \frac{\left(1 - \frac{v^2}{c^2}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \tag{8}$$

or:

$$\mathrm{d}t_{AC} = \frac{\mathrm{d}t_{BC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{9}$$

$$\mathrm{d}t_{BC} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \cdot \mathrm{d}t_{AC} \tag{10}$$

Equations (9) and (10) are obviously identical, and are equal to the equations in the quotation from Koobee Wublee's text above, if we ignore the unnecessary renaming of dt_{AC} to dt_{AB} and dt_{BC} to dt_{BB}.

Both equations state that since a clock at C is moving at the speed v in frame A it will appear to run slow as observed in frame A.

Quotation from Koobee Wublee's text continues:

> However, this does not prevent the ilks like Paul Andersen to play mathemagic tricks to lie about the twin paradox. What Paul did is to flip dt_{BB} into dt_{BA} and dt_{AB} into dt_{AA} in the reciprocal equation as describe below. <shrug>
>
> $**$ $dt_{AB} = dt_{BB}/\sqrt{1 - v_{BA}^2/c^2}$, main
>
> And
>
> $**$ $dt_{BA} = dt_{AA}\sqrt{1 - v_{AB}^2/c^2}$, reciprocal, Pauls mathemagics
>
> Where
>
> $**$ dt_{BB} mathemagically turns into dt_{BA} in the reciprocal equation
> $**$ dt_{AB} mathemagically turns into dt_{AA} in the reciprocal equation

Quotation ends

My comment:

This seems rather confused. I will however play the mathemagic trick to flip dt_{AC} into dt_{BC} and dt_{BC} into dt_{AC}.

If C is stationary in frame A we have $\frac{\mathrm{d}x_{AC}}{\mathrm{d}t_{AC}} = 0$ and $\frac{\mathrm{d}x_{BC}}{\mathrm{d}t_{BC}} = v$. Inserting these in equations (3) and (4) yields:

$$\frac{\mathrm{d}t_{AC}}{\mathrm{d}t_{BC}} = \frac{\left(1 - \frac{v^2}{c^2}\right)}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \tag{11}$$

$$\frac{\mathrm{d}t_{BC}}{\mathrm{d}t_{AC}} = \frac{1}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{12}$$

or:

$$\mathrm{d}t_{AC} = \sqrt{\left(1 - \frac{v^2}{c^2}\right)} \cdot \mathrm{d}t_{BC} \tag{13}$$

$$\mathrm{d}t_{BC} = \frac{\mathrm{d}t_{AC}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \tag{14}$$

Equations (13) and (14) are obviously identical.

Both equations state that since that since a clock at C is moving at the speed v in frame B it will appear to run slow as observed in frame B.

Conclusion

We do not really need the object C.

When A and B are two frames of reference in relative motion we can state:
A stationary clock in frame A will appear to run slow when observed in frame B.
A stationary clock in frame B will appear to run slow when observed in frame A.

This has nothing to do with the twin paradox. It is mutual time dilation.