The prediction of fringe shifts in Michelson & Morley's repetition of Fizeau's experiment

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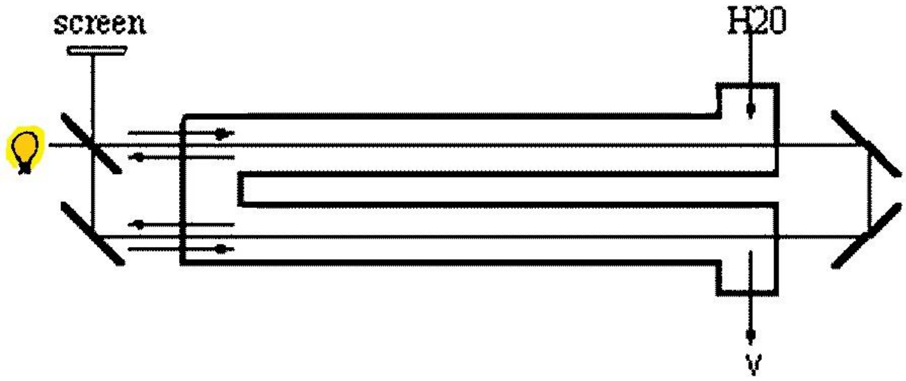

Figure 1: *The Fizeau interferometer*

The figure above shows the interferometer used in Fizeau's original experiment.
The interferometer used by Michelson & Morley was a little different, but the principle remains the same.

Michelson and Morley's original paper can be seen here:
Influence of Motion of the Medium on the Velocity of Light

Generally, there are two ways of calculating the phase difference between the two beams in an interferometer:

1. The difference in transit time for the two beams can be calculated. The phase difference is then $\Delta\varphi = 2\pi\nu \cdot \Delta t = (2\pi c/\lambda)\Delta t$ where Δt is the difference in transit time, ν is the frequency of the light, λ is the wavelength in vacuum, and c is the speed of light in vacuum.

2. The number of wavelengths in the two beams can be compared. The phase difference is then $\Delta\varphi = 2\pi\Delta N$ where ΔN is the difference in the number of wavelengths in the two beams.

Even if the two methods necessarily must give the same result, we will calculate the predictions both ways.

Calculation of the fringe shift by difference in transit times

- Let $c_f = \dfrac{c}{n} + v \cdot x$ be the speed of the light in the ray that is moving with the water, as measured in the lab frame.
- Let $c_b = \dfrac{c}{n} - v \cdot x$ be the speed of the light in the ray that is moving in the opposite direction, as measured in the lab frame.
- Let L be the length of each beam as measured in the lab frame.

$$\Delta t = \frac{L}{c_b} - \frac{L}{c_f} = L\left(\frac{c_f - c_b}{c_f c_b}\right) = \frac{2Lvn^2x}{c^2\left(1 - \left(\frac{nvx}{c}\right)^2\right)} \tag{1}$$

We can ignore the second order terms in v/c, and get:

$$\Delta t \approx \frac{2Lvn^2x}{c^2} \tag{2}$$

The phase difference is:

$$\Delta\varphi = 2\pi\nu\Delta t = \frac{2\pi c}{\lambda}\Delta t \approx \frac{4\pi Lvn^2x}{\lambda c} \tag{3}$$

The fringe displacement Δ is the displacement of a fringe relative to the inter fringe distance.

$$\Delta = \frac{\Delta\varphi}{2\pi} \tag{4}$$

Michelson and Morley measured the displacement of the fringes when they reversed the speed of the water. So the total fringe displacement will be:

$$\Delta_{tot} = 2 \cdot \frac{\Delta\varphi}{2\pi} \approx \frac{2Lvn^2x}{\lambda c} \tag{5}$$

Calculation of the fringe shift by difference in number of wavelengths

- Let ν be the frequency of the light source as measured in the lab frame.
- Let λ be the wavelength in vacuum of light with frequency ν, $\lambda = \dfrac{c}{\nu}$
- Let λ_f be the wavelength of the ray that is moving with the water, as measured in the lab frame.
- Let λ_b be the wavelength of the ray that is moving in the opposite direction, as measured in the lab frame.

The wavelength of the beam going with the water will be:

$$\lambda_f = \frac{c_f}{\nu} = \frac{\lambda}{c}\, c_f = \frac{\lambda}{c}\left(\frac{c}{n} + vx\right) \tag{6}$$

The wavelength of the beam going in the opposite direction will be:

$$\lambda_b = \frac{c_b}{\nu} = \frac{\lambda}{c}\, c_b = \frac{\lambda}{c}\left(\frac{c}{n} - vx\right) \tag{7}$$

The number of wavelengths in the beams will be:

$$N_f = \frac{L}{\lambda_f} = \frac{L}{\frac{\lambda}{c}\left(\frac{c}{n} + vx\right)} = \frac{Lc}{\lambda\left(\frac{c}{n} + vx\right)} \tag{8}$$

$$N_b = \frac{L}{\lambda_f} = \frac{L}{\frac{\lambda}{c}\left(\frac{c}{n} - vx\right)} = \frac{Lc}{\lambda\left(\frac{c}{n} - vx\right)} \tag{9}$$

The difference in the number of wavelengths will be:

$$\Delta N = N_b - N_f = \frac{Lc}{\lambda\left(\frac{c}{n} - vx\right)} - \frac{Lc}{\lambda\left(\frac{c}{n} + vx\right)} = \frac{2Lvn^2x}{\lambda c\left(1 - \left(\frac{nvx}{c}\right)^2\right)} \tag{10}$$

We can ignore the second order terms in v/c, and get:

$$\Delta N \approx \frac{2Lvn^2x}{\lambda c} \tag{11}$$

Michelson measured the fringe displacement when the water flow was reversed. So the total fringe shift will be:

$$\Delta_{tot} = 2 \cdot \Delta N \approx \frac{4Lvn^2x}{\lambda c} \tag{12}$$

which is the same result as above.

Michelson's fringe shift

If we assume that the speed of light in moving water can be written:

$$c_{f,b} = \frac{c}{n} \pm v \cdot x \tag{13}$$

then the fringe displacement in Michelson & Morley's repetition of Fizeau's experiment will be:

$$\Delta_{tot} = \frac{4Lvn^2x}{\lambda c} \tag{14}$$

The 'drag coefficient' x is thus:

$$x = \frac{\lambda c}{4Lvn^2} \cdot \Delta_{tot} \tag{15}$$

Michelson & Morley measured: $x = 0.434 \pm 0.02$

Prediction of The Special Theory of Relativity

The speed of light in the frame of reference where the water is stationary is $\frac{c}{n}$ according to SR. Transformed to the lab frame, we get:

$$c_f = \frac{\frac{c}{n} + v}{1 + \frac{\frac{c}{n} \cdot v}{c^2}} \approx \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right) \tag{16}$$

$$c_b = \frac{\frac{c}{n} - v}{1 - \frac{\frac{c}{n} \cdot v}{c^2}} \approx \frac{c}{n} - v\left(1 - \frac{1}{n^2}\right) \tag{17}$$

That is, SR predicts: $x = \left(1 - \frac{1}{n^2}\right) = 0.438$, which is about in the middle of the error bars. (The error is $< 1\%$, the error bars are $\pm 4.6\%$)